

Computers, brains and AI

2: Foundations



Today...

Binary

Numbers

Logic



Binary



on / off

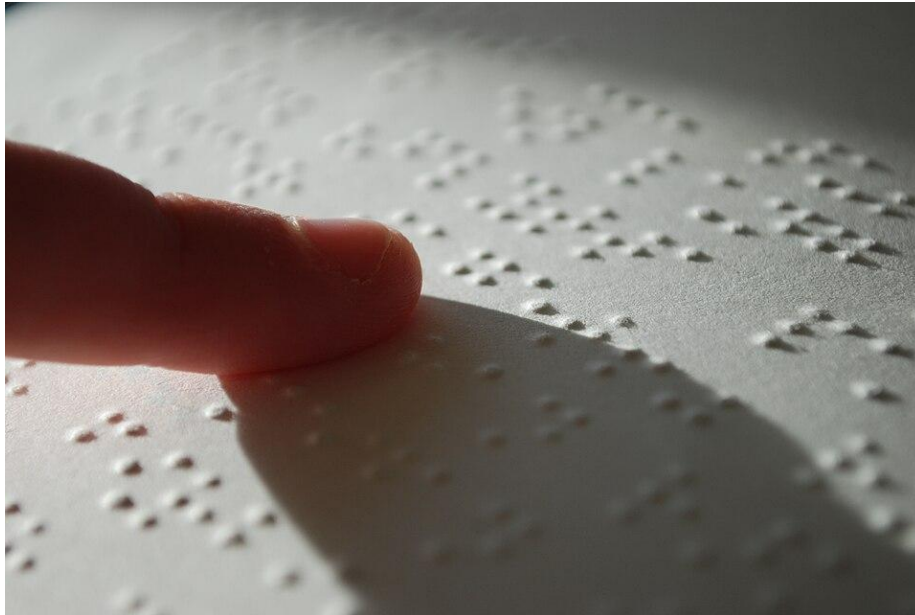
0 / 1

yes / no

true / false

"Bit" is short for
binary digit.

The "bi" in "binary"
means two states.



A	●	■			
B	■	■	●	●	●
C	■	■	●	■	●
D	■	■	●	●	
E	●				

1824

Binary data has been
around for a long time.

1837



E •
T —

Morse code

Z •••••
•••••
•••••
•••

Not Morse code

I ••
A • —
N — •
M — —

S • ••
U • • —
R • — •
W • — —

D — ••
K — • —
G — — •
O — — —

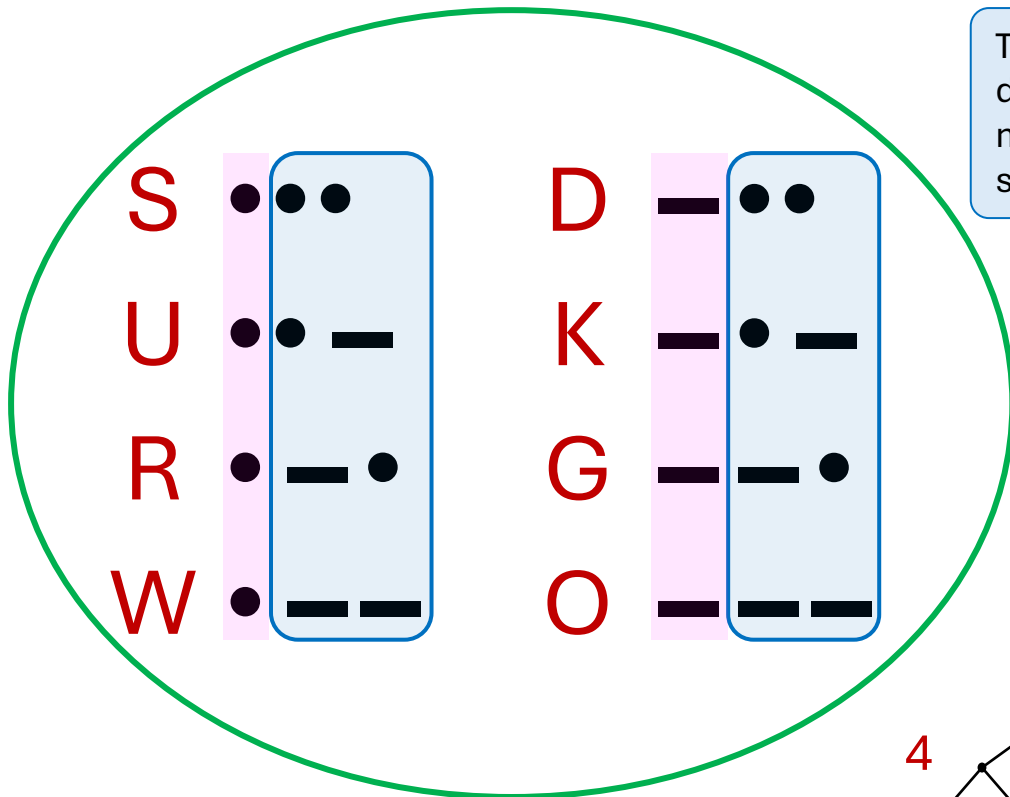
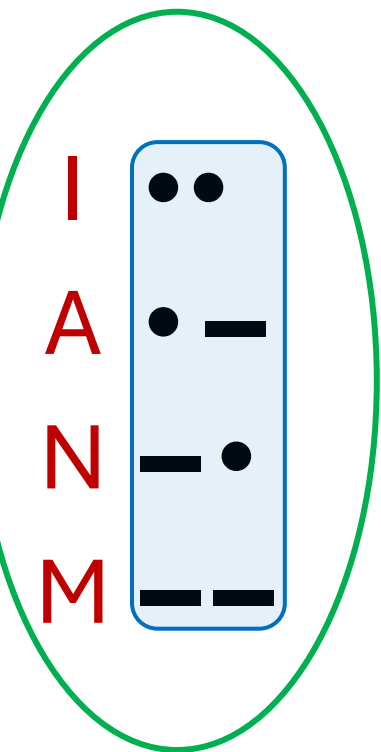
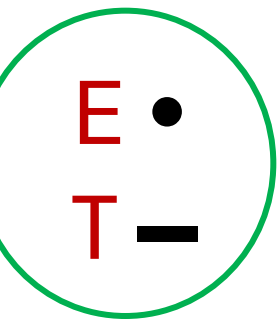
A • B •• C •••
D •••• E •••••
Z •••••
•••••
•••••
•••

People instinctively know that two bits can encode four different possibilities.

This trick shows that 3 bits can encode exactly 8 different possibilities.

I II III





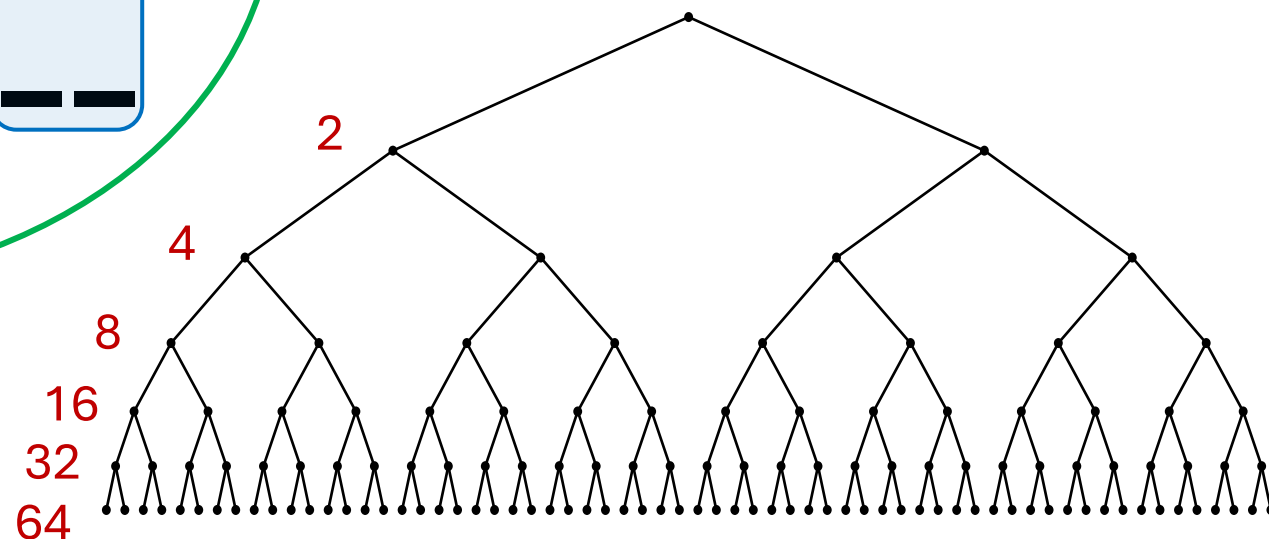
Morse code

The number of possibilities doubles for every new bit, making the total grow surprisingly quickly.

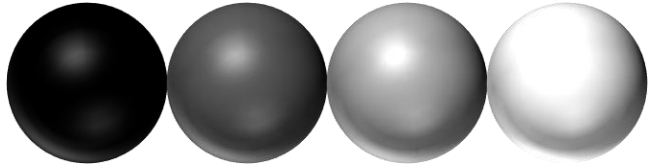
1	2
2	4
3	8
4	16

30

1,073,741,824



00 01 10 11



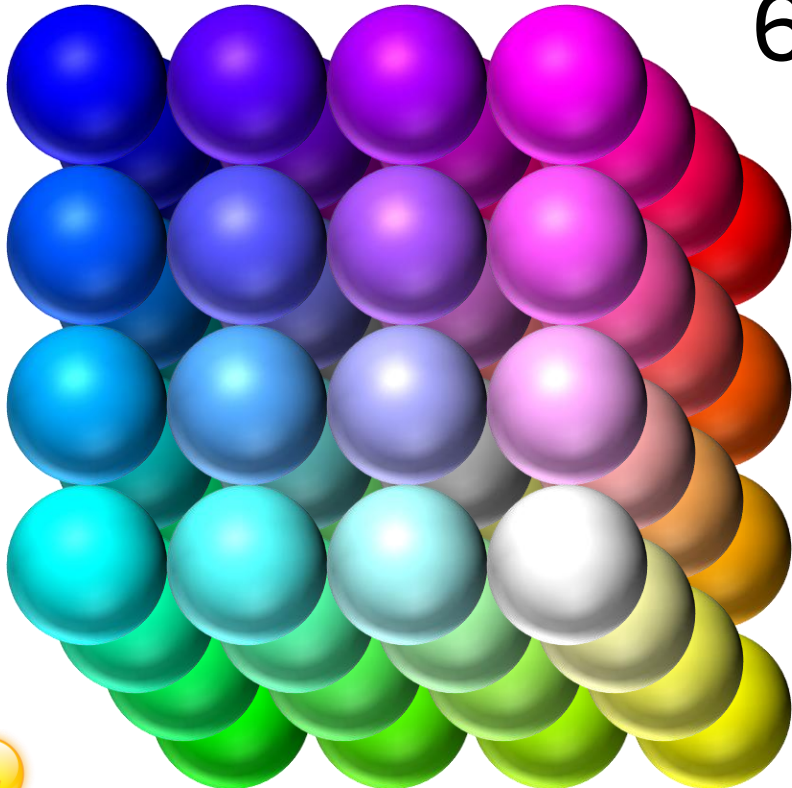
2 bits

Humans can distinguish about 8 million colours, and so 24 bits (8 bits for red, 8 bits for green and 8 bits for blue) is enough for displaying any colour.

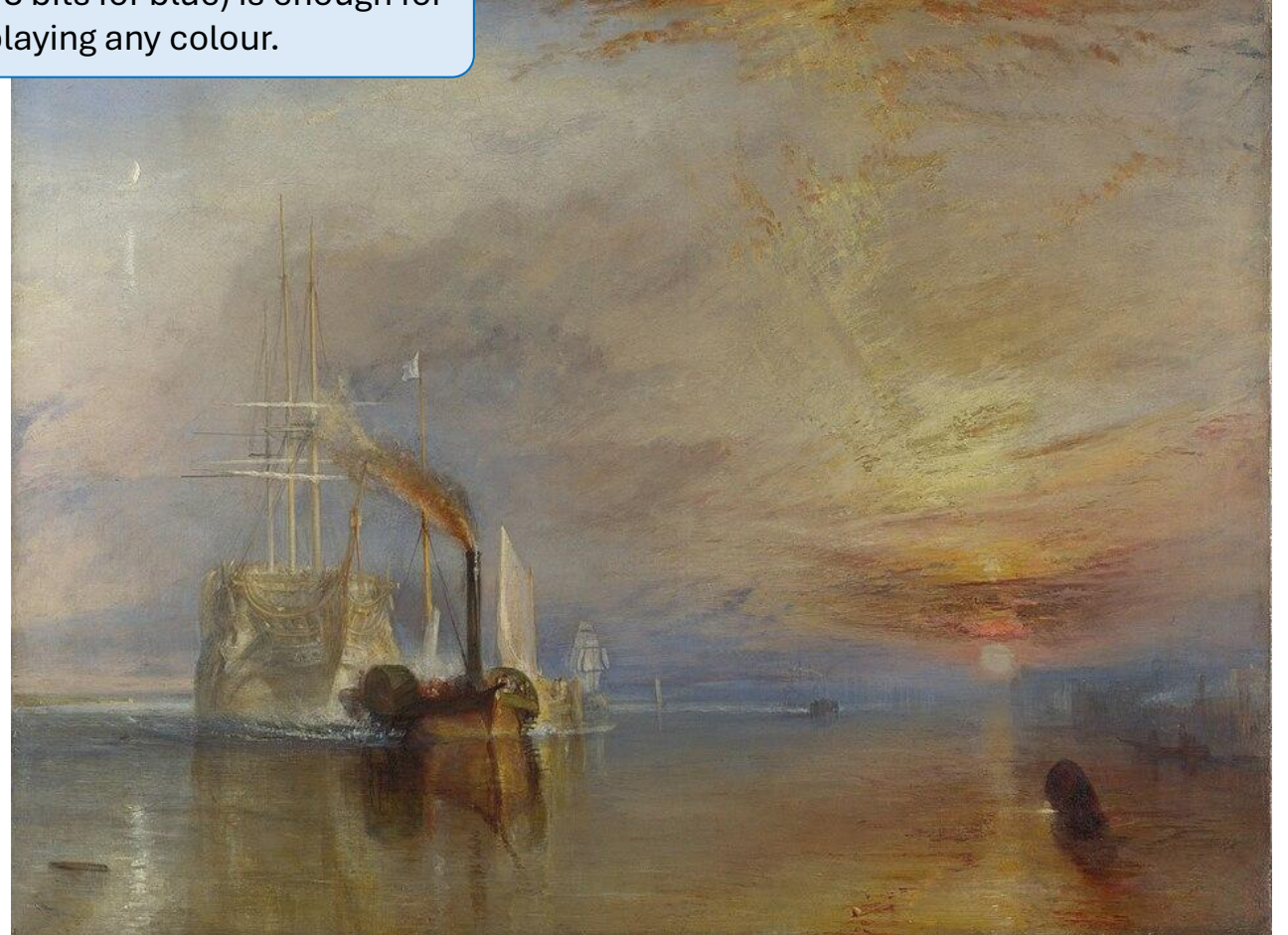
24 bits

?

With just 6 bits we can specify 64 different colours.



6 bits



16 million possible colours



森

0110 1000 1110 1110

β

0000 0011 1011 0010

ê

0000 0000 1110 1010

ℵ

0000 0101 1101 0000

𐂔

0011 0000 0111 0110

Unicode

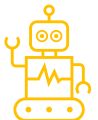
16 65,536

Unicode is an international agreement on how to specify all the world's characters.

With just 16 bits, there are 65,536 possibilities.

Being binary doesn't (necessarily) make computers and robots boring.

https://en.wikipedia.org/wiki/List_of_Unicode_characters



Big numbers



intuition

Powers of 10

1,000,000

100,000

This number scale is pleasing,
as well as useful. It has nice,
round numbers and a large
dynamic range.

10,000

1,000

3rd power

10^3

$10 \times 10 \times 10$

100

2nd power

10^2

10

1st power

10^1

1



Powers of 10

1,000,000

100,000

10,000

1,000

100

10

1

3rd power

2nd power

1st power

10^3

10^2

10^1

1,000

log

3

The log (to base 10) of 1000 is 3.

Logs are the opposite of powers



log scale



Richter scale Decibels

100,000

For a small child,
a natural halfway
point might be 3.

1 3 9

$\times 3$

$\times 3$

[https://en.wikipedia.org/
wiki/Logarithm#Music](https://en.wikipedia.org/wiki/Logarithm#Music)

Log scales come up
anywhere there is a
large range of values.

10,000

exponent

exponential

log

1,000

3rd power

10³

1,000

3

100

2nd power

10²

A logarithmic spiral

10

1st power

1

When the ancient Greek astronomers classified stars by
brightness, they accidentally used a log scale (which
turned out to be very handy for modern astronomy
because of the huge range of brightnesses).

[https://en.wikipedia.org/wiki/Magnitude_\(astronomy\)](https://en.wikipedia.org/wiki/Magnitude_(astronomy))



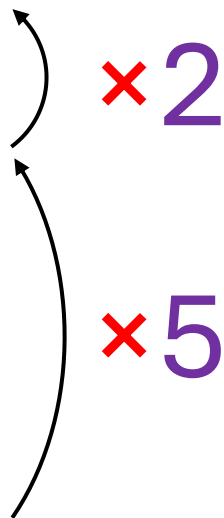
100

50

10

5

1



Moore's Law

Transistor count

50,000,000,000

10,000,000,000

5,000,000,000

1,000,000,000

500,000,000

100,000,000

50,000,000

10,000,000

5,000,000

1,000,000

500,000

100,000

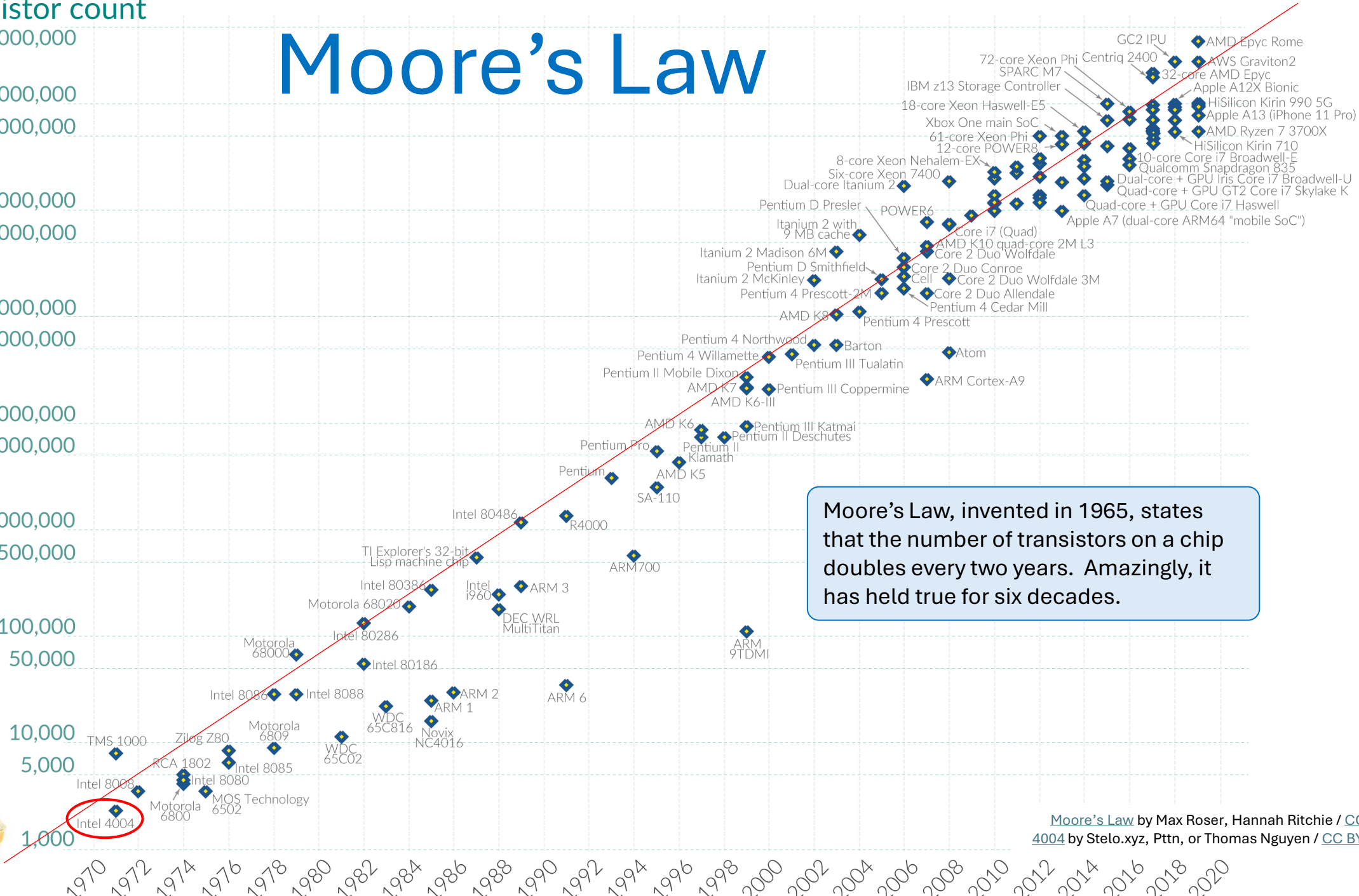
50,000

10,000

5,000

1,000

Number of
transistors
(switches)



Moore's Law, invented in 1965, states that the number of transistors on a chip doubles every two years. Amazingly, it has held true for six decades.

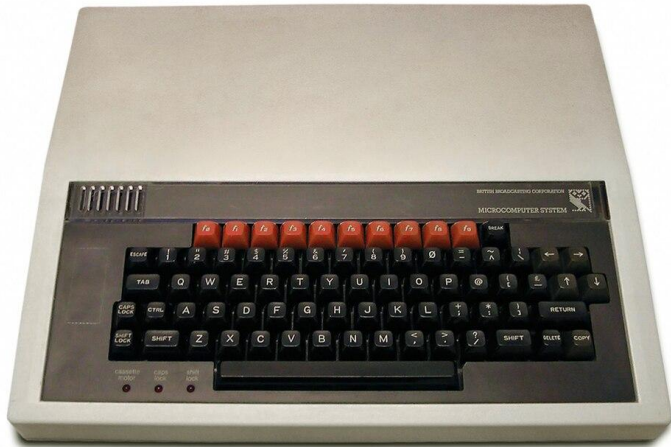
Moore's Law by Max Roser, Hannah Ritchie / CC BY 4.0
4004 by Stelo.xyz, Pttm, or Thomas Nguyen / CC BY-SA 4.0

1,000



BBC Micro

1981



6502

Sinclair Spectrum

1982



Z80

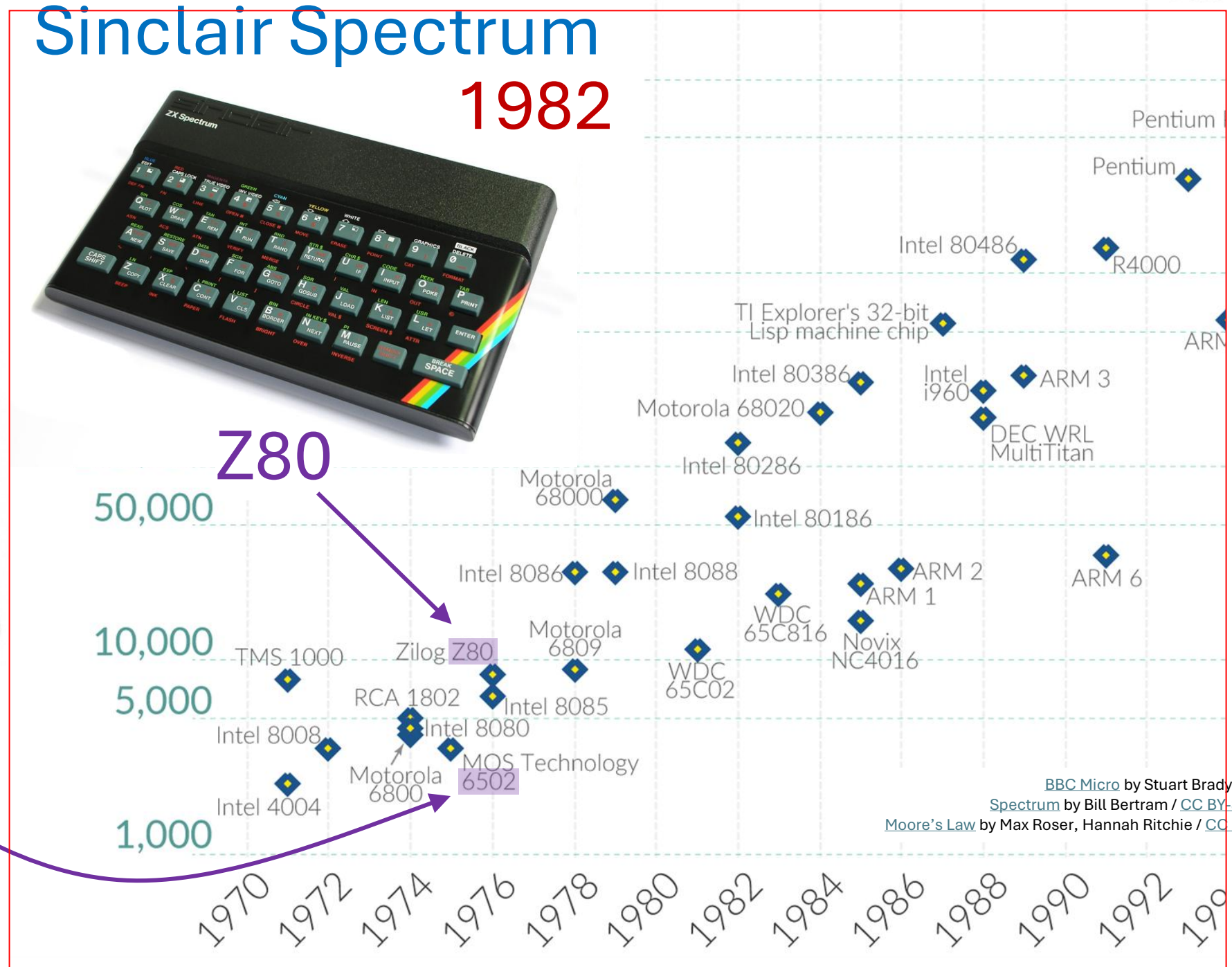
50,000

10,000

5,000

1,000

1970 1972 1974 1976 1978 1980 1982 1984 1986 1988 1990 1992 1994



BBC Micro by Stuart Brady / CC0
Spectrum by Bill Bertram / CC BY-SA 2.5
Moore's Law by Max Roser, Hannah Ritchie / CC BY 4.0



Transistor count

Moore's Law

50,000,000,000

10,000,000,000

5,000,000,000

1,000,000,000

500,000,000

100,000,000

50,000,000

10,000,000

5,000,000

1,000,000

500,000

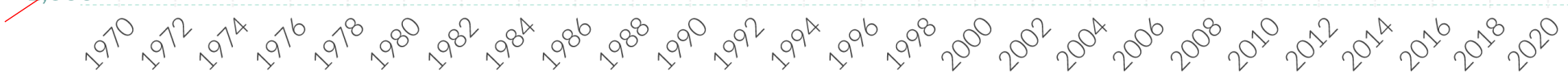
100,000

50,000

10,000

5,000

1,000

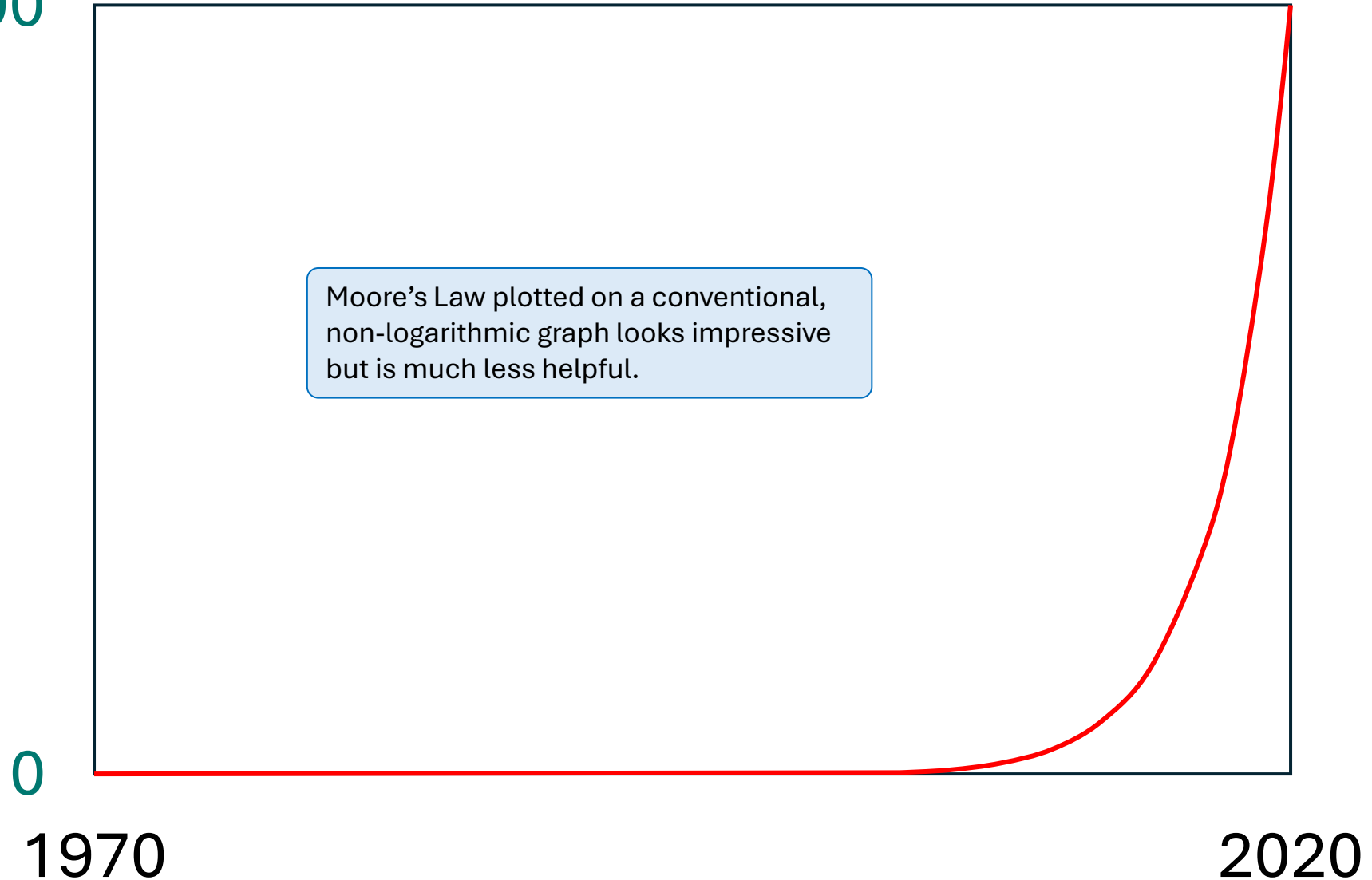


Our World
in Data

Moore's Law by Max Roser, Hannah Ritchie / CC BY 4.0

Transistor count

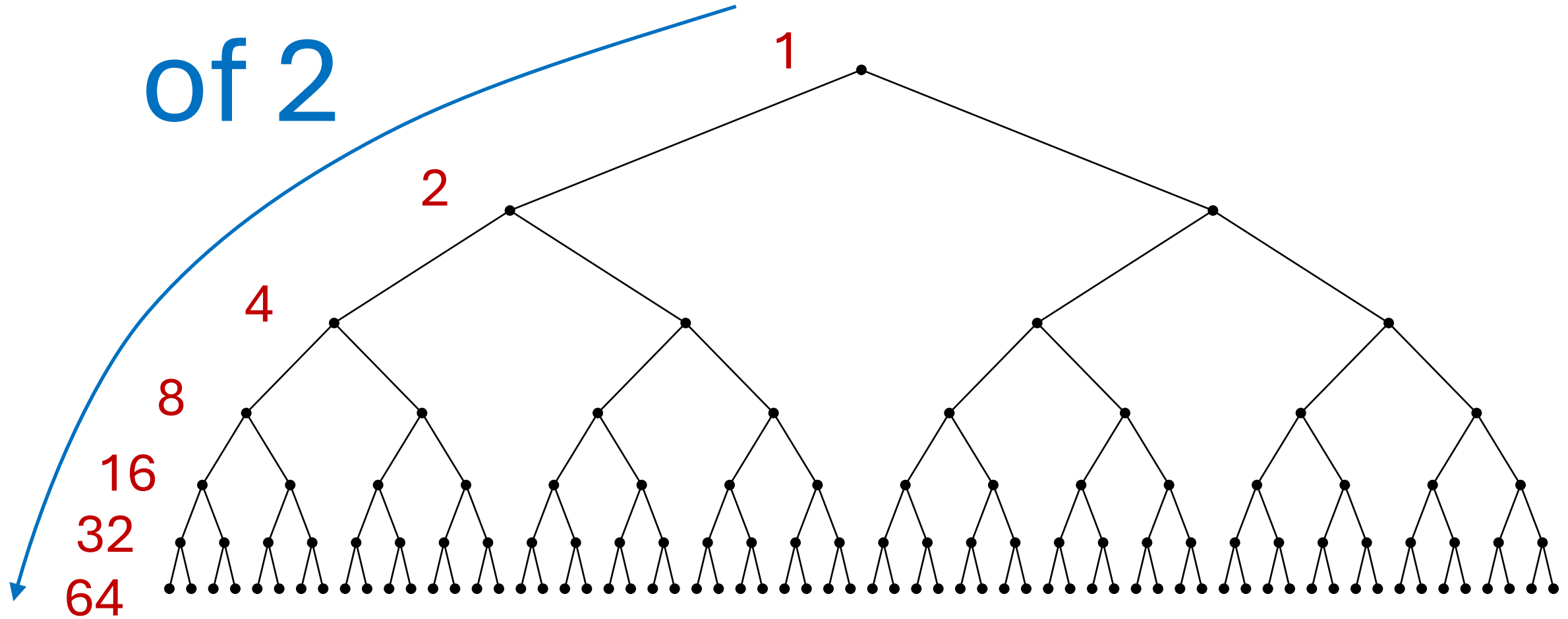
50,000,000,000

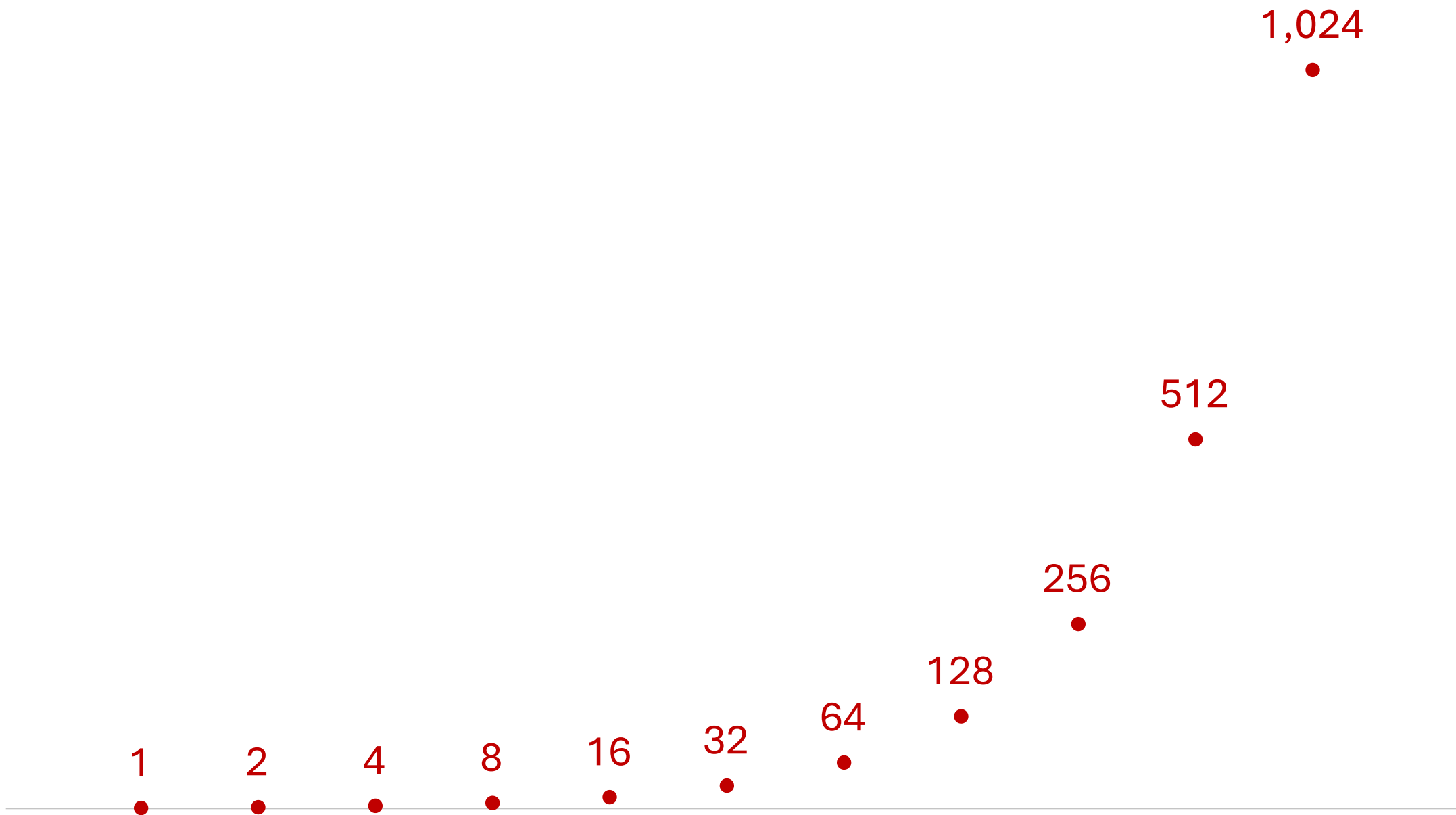


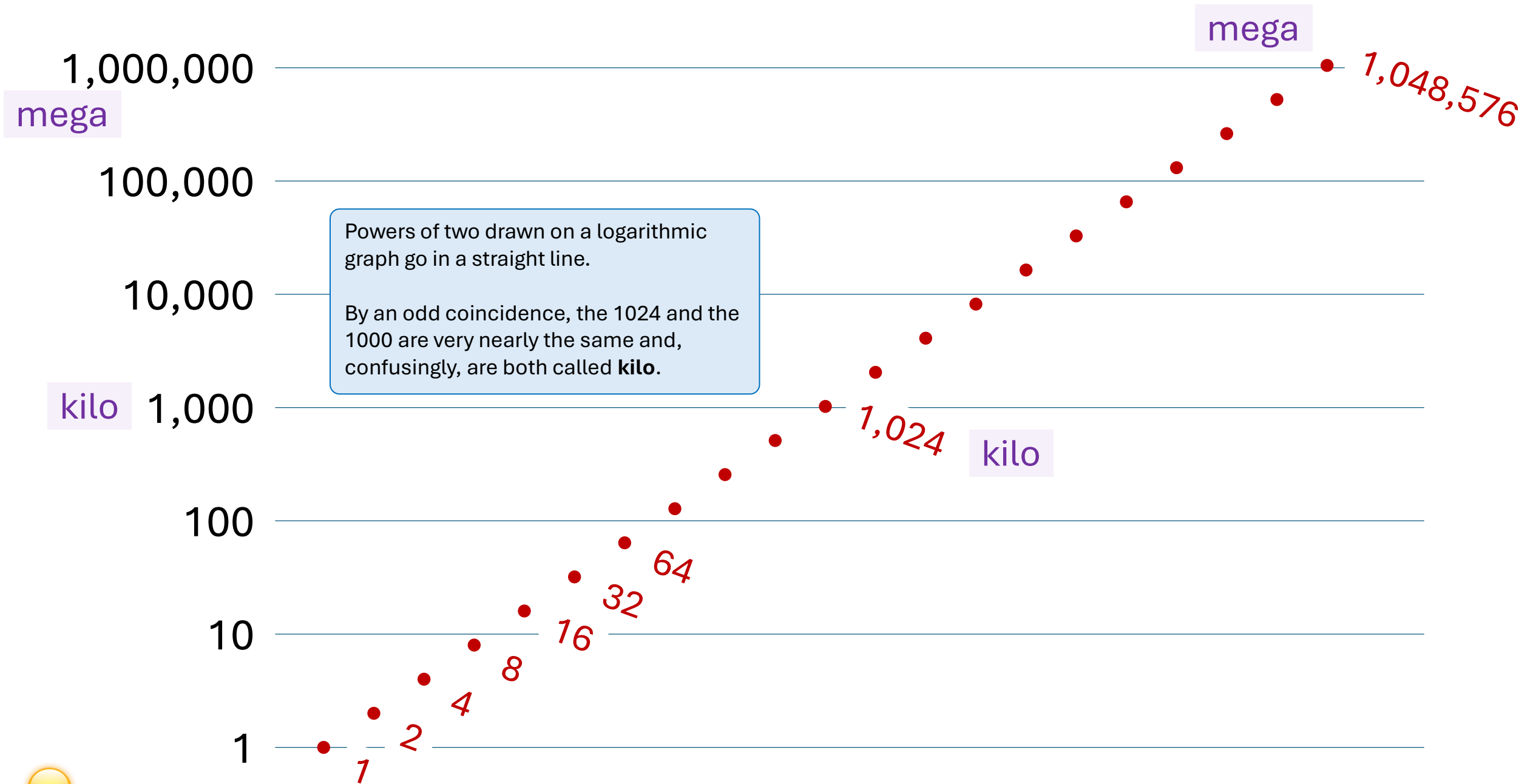
Moore's Law



powers of 2







1,000,000,000,000,000,000
100,000,000,000,000,000
10,000,000,000,000,000
1,000,000,000,000,000
100,000,000,000,000
10,000,000,000,000
1,000,000,000,000
100,000,000,000
10,000,000,000
1,000,000,000
100,000,000
10,000,000
1,000,000
100,000
10,000
1,000
100
10
1

The sheer number of zeros
make us go cross-eyed.

Therefore, we need a better
notation, as shown on the
next slide.

quintillion

1,000,000,000,000,000,000

quadrillion

1,000,000,000,000,000

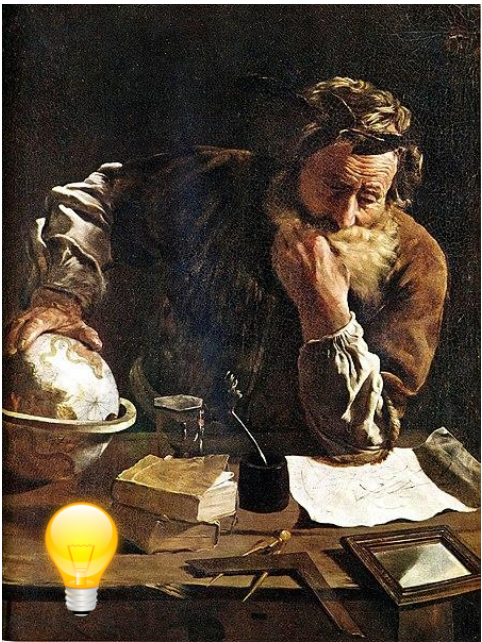
trillion

1,000,000,000,000

billion

1,000,000,000

Archimedes



Archimedes had a similar system for very large numbers.

1,000,000

1,000

1

— 10^{18}

— 10^{15} 1015

— 10^{12}

— 10^9

— 10^6

— 10^3 — $10 \times 10 \times 10$ —

— 10^0 —

~~10 raised to the 15th power~~

10 to the 15

10 to the 15

If you ever see a number slightly larger than a thousand, it might be, say, 10^{15} being mistyped as 1015.

When said aloud, the “to” and the “the” are barely audible. But you can’t just say “ten fifteen” because the rhythm would be wrong.

floating-point

10^{18} operations per second (10^{18} flops)

+ - × ÷

2

3

3.65

36.5

Floating-point numbers can have fractional values like 3.5, as well as whole number values like 3 or 4.

They are used in scientific simulations and in large language models (as we will see in later lectures).

Frontier was the fastest computer in the world (until it was overtaken by **El Capitan** in November 2024).

Although it uses 22 megawatts, it held the record for the most efficient supercomputer, measured in flops per watt.

22 MW

Human brain: 20 W



Cray-1

1975

160 MFLOPS

6 *billion times*
slower

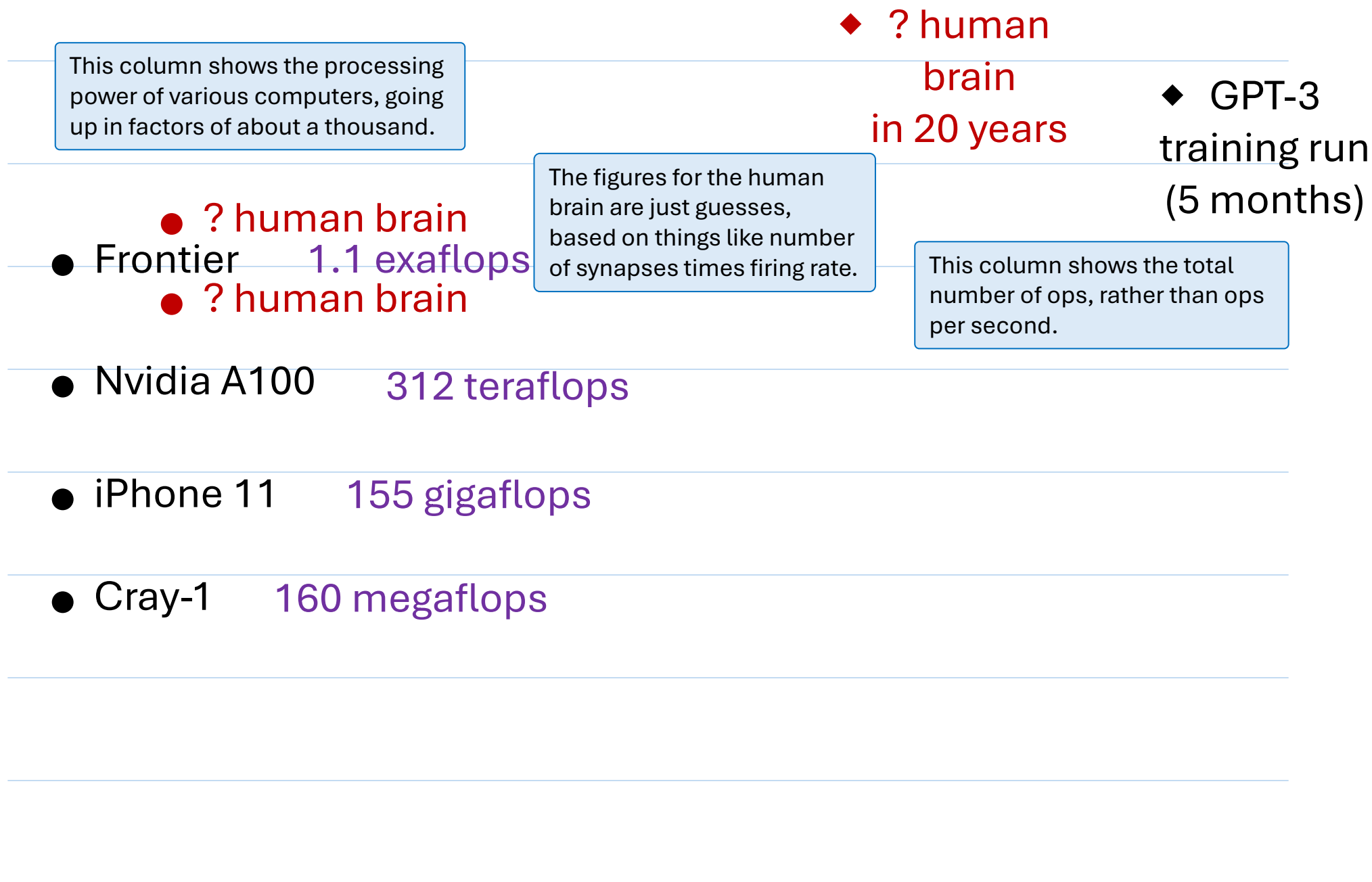
The **Cray-1** was by far the fastest computer in the world in 1975 but was 6 billion times slower than **Frontier**.

115 kW

30 million times less
energy efficient

It also had 30 million times
fewer flops per watt than
Frontier.







10^{160}

permutations of two packs of cards

10^{140}

In the game of big numbers, both astronomy and computers are dwarfed by simple things like the number of permutations of a pack of cards.

10^{120}

the observable universe as a computer

...assuming each atom somehow does a trillion ops per second for ten billion years.

10^{100}

10^{80}

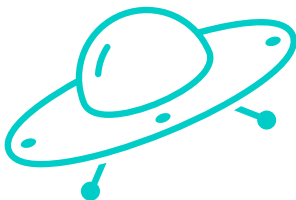
atoms in the observable universe

permutations of a pack of cards

10^{60}

https://en.wikipedia.org/wiki/Fermi_paradox

Where are all the aliens?



10^{40}

10^{20}

stars in the observable universe

GPT-3 training run

all the computers for a year

1

Binary numbers

0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Binary



Leibniz

We can list the binary numbers by repeating the patterns we saw earlier...

...and then putting zeros or ones in front.

I	••
A	•—
N	—•
M	— —

4

S	•••	D	—••
U	••—	K	—•—
R	•—•	G	— —•
W	•— —	O	— — —

8

4	2	1	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Binary



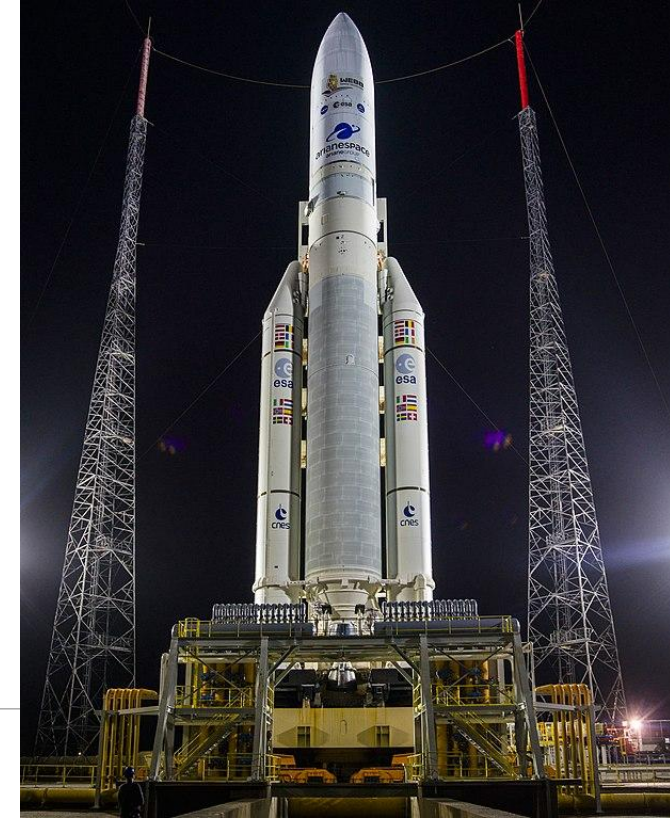
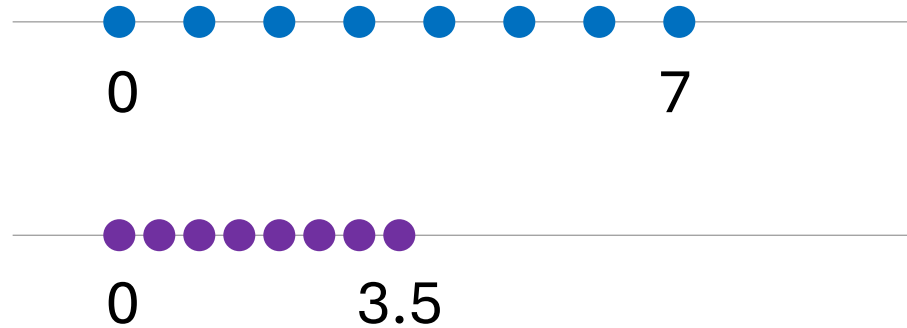
Leibniz



0	0	.	0	0
0	0	.	1	0.5
0	1	.	0	1
0	1	.	1	1.5
1	0	.	0	2
1	0	.	1	2.5
1	1	.	0	3
1	1	.	1	3.5

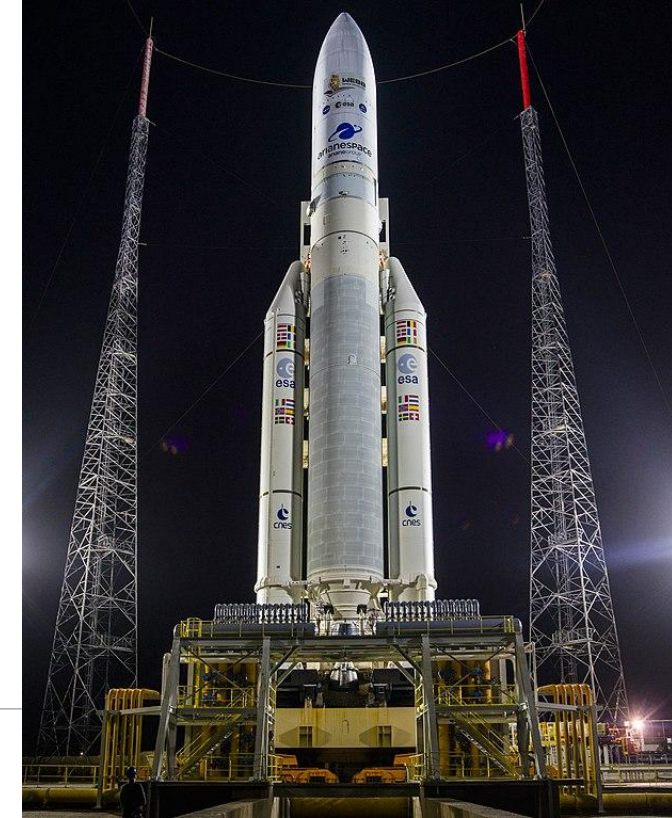
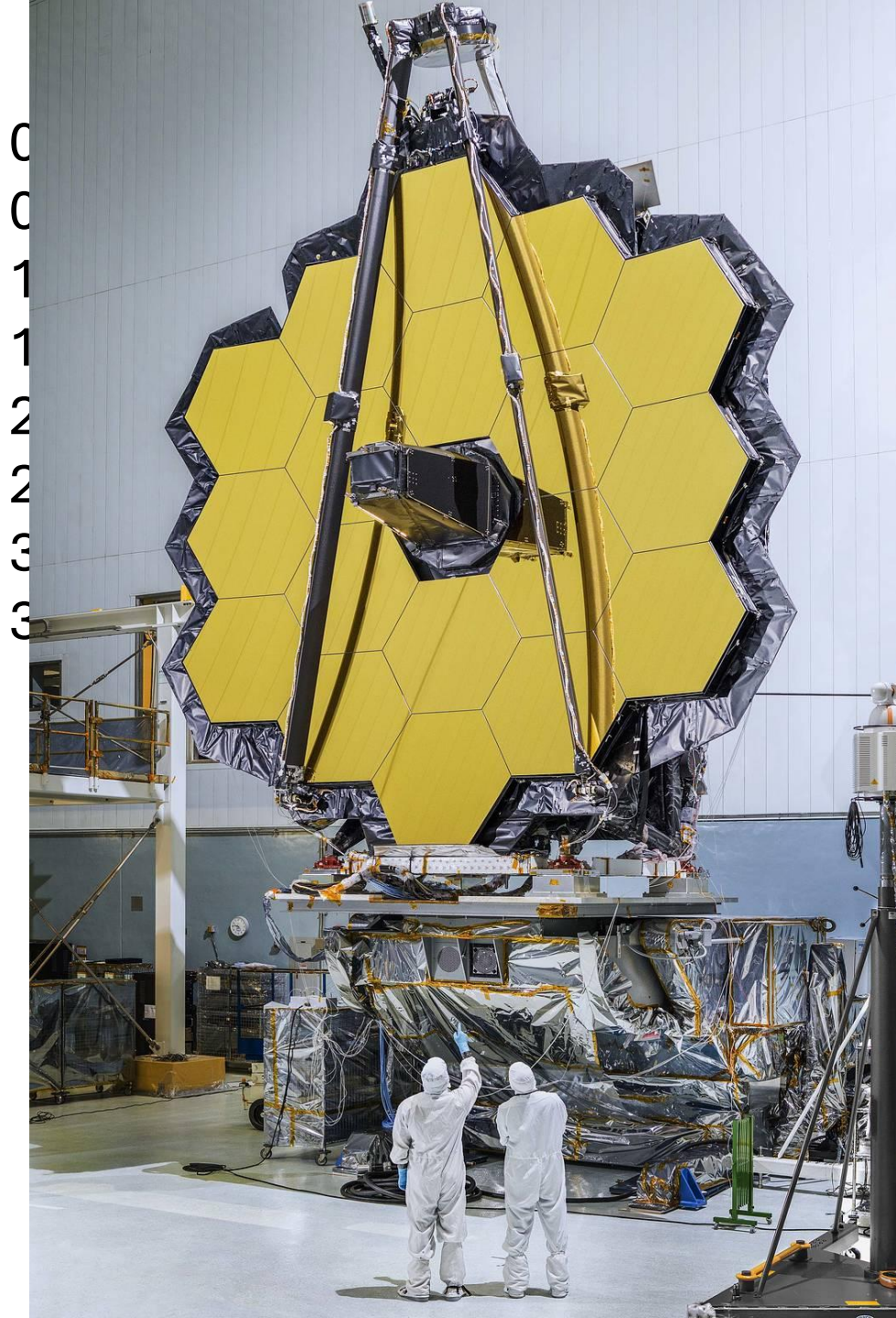
Fixed-point numbers can represent fractions. (Floating-point numbers are another way of getting fractions).

Fixed-point numbers



Ariane 5

0	0	.	0
0	0	.	1
0	1	.	0
0	1	.	1
1	0	.	0
1	0	.	1
1	1	.	0
1	1	.	1



Ariane 5

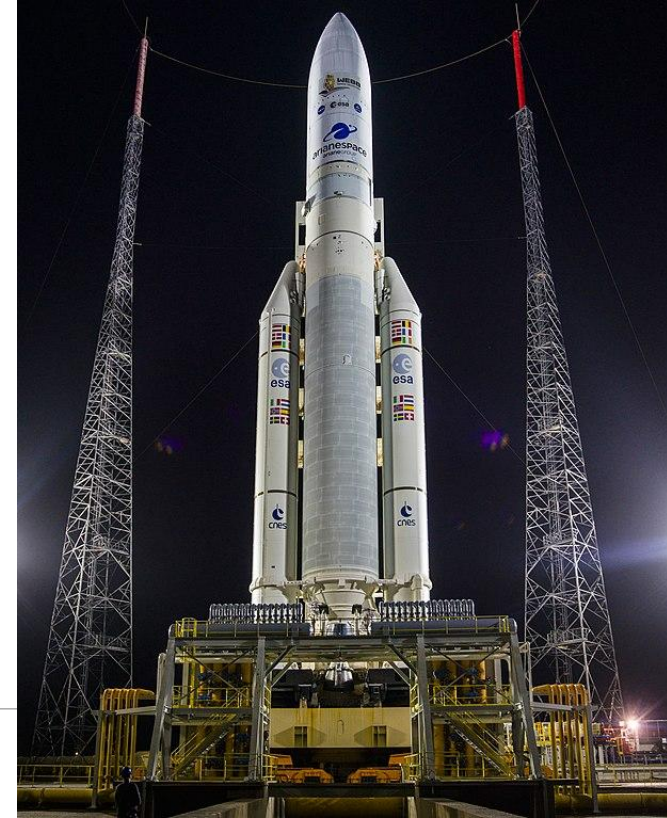
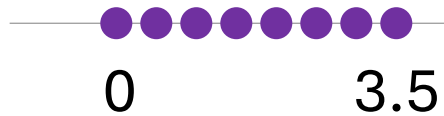
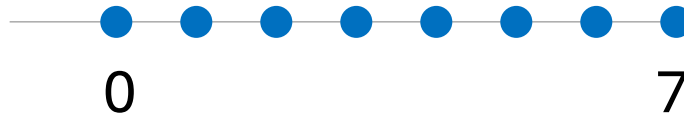
The maiden voyage of the otherwise brilliant Ariane 5 crashed because of a numerical overflow.

Among other things, it successfully put the James Webb Space Telescope into orbit far beyond the moon.

0	0	.	0
0	0	.	1
0	1	.	0
0	1	.	1

0
0.5
1
1.5
2
2.5
3
3.5

Fixed-point numbers



Ariane 5

Negative numbers

Sign bit

Here are two ways to represent negative numbers in binary.

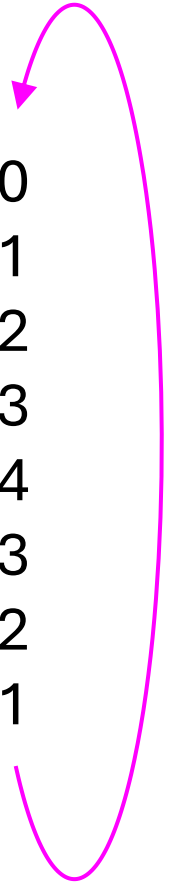
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

+

-

0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	-0
1	0	1	-1
1	1	0	-2
1	1	1	-3

0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	-4
1	0	1	-3
1	1	0	-2
1	1	1	-1



Why binary?

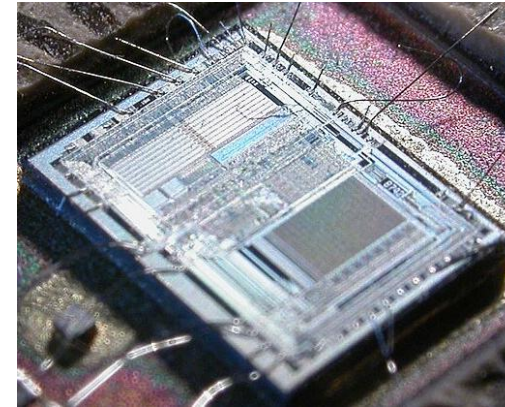
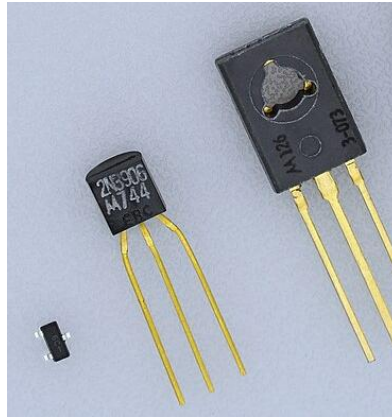
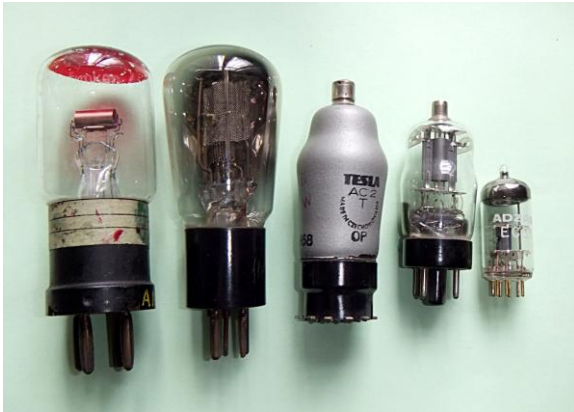


Decimal



Nearly all computers are binary.

Binary



Digital

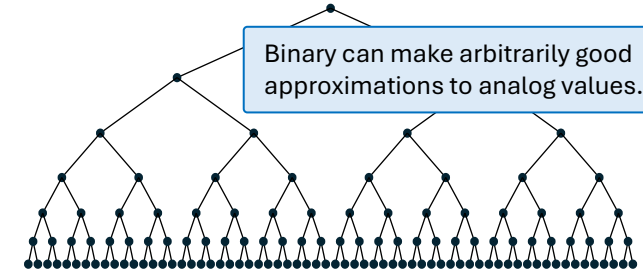
[Click this link to see a transistor.](#)

Transistor

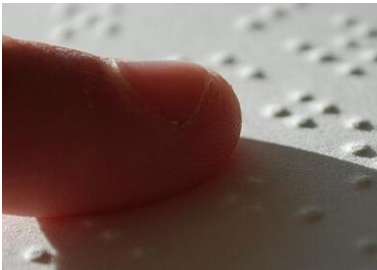
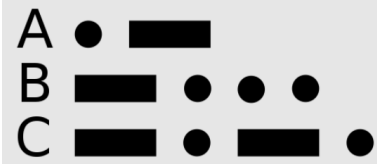
A transistor (even a transistor in a digital computer) can behave in an analog fashion, but computers are designed so that transistors are usually fully on or fully off.

Analog

Binary can make arbitrarily good approximations to analog values.



Binary



Ternary



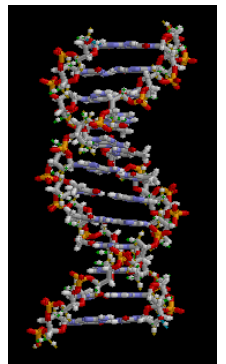
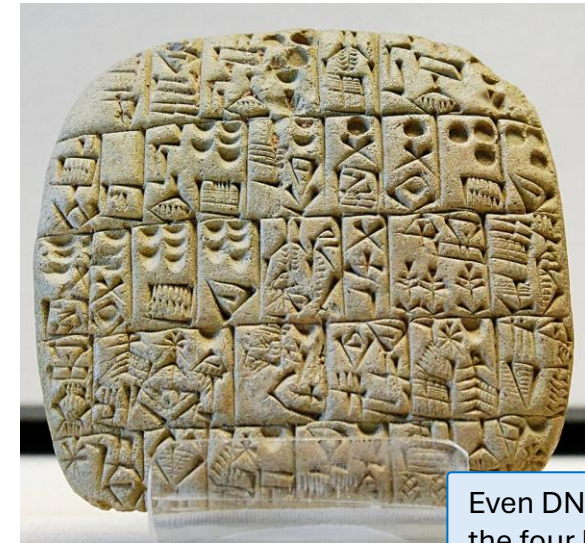
Morse code could have used 3 values, instead of 2, but it would have been awkward and error prone.

Most things in modern life seem to be digital. Digital values are convenient because they can be copied exactly.

Decimal



Digital things



Even DNA is digital, since it uses the four letters, C, G, A and T.



[Money](#) by Avij

[DNA](#) by brian0918

[Dice](#) by Brett Jordan / [Unsplash](#)

[Cuneiform](#) by Unknown artist - Marie-Lan Nguyen

Binary

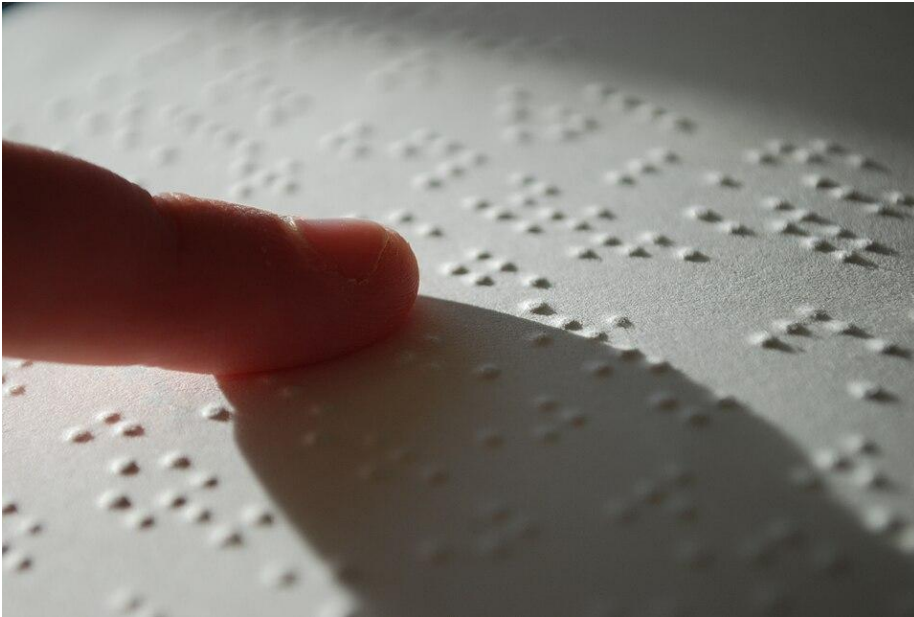
bit

on / off

0 / 1

yes / no

true / false



1824

A	●	■			
B	■	●	●	●	
C	■	●	■	●	
D	■	●	●		
E	●				

1837



logic

These links show different electronic circuits.

The links with the larger font were the ones we saw in the lecture.

All the links take you to the amazing <https://falstad.com/circuit/circuitjs.html>, where you can draw circuits of your own or choose from their circuits in the Circuits pulldown menu.

Logic

Arithmetic

You can do arithmetic with logic gates...

?

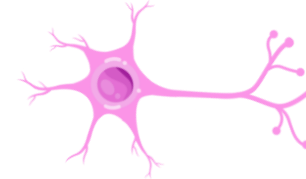
...and memory.



Transistor
NOT gate

Two transistors

A logic gate is like a simple neuron.



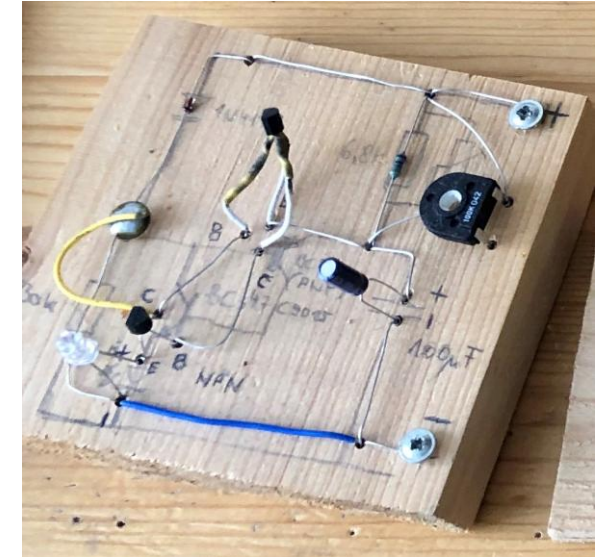
AND gate

AND gates

Making an AND gate

XOR gate OR gate

NAND gate



A microprocessor is mostly made of logic gates.

Majority vote

Full 1-bit adder

3-bit adder

Round and round

Reset

Transistors