

Computers, brains and AI

2: Foundations



Today...

Binary

Numbers

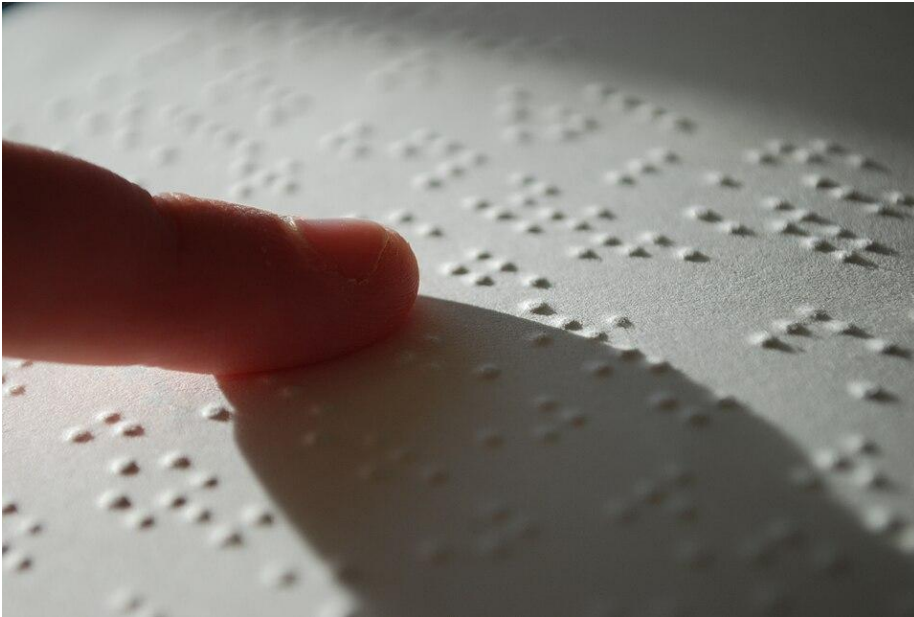
Logic



Binary

bit

on / off
0 / 1
yes / no
true / false



1824

A	●	▬			
B	▬	●	●	●	
C	▬	●	▬	●	
D	▬	●	●		
E	●				

1837



E •
T —

Morse code

Z •••••
•••••
•••••
•••

Not Morse code

I ••
A • —
N — •
M — —

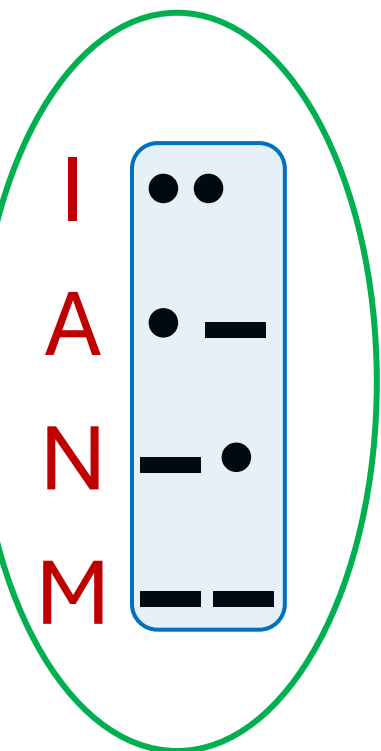
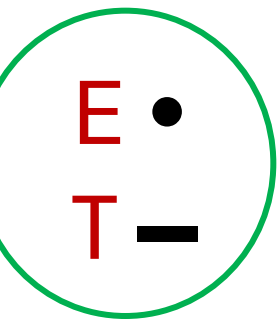
S • ••
U • • —
R • — •
W • — —

D — ••
K — • —
G — — •
O — — —

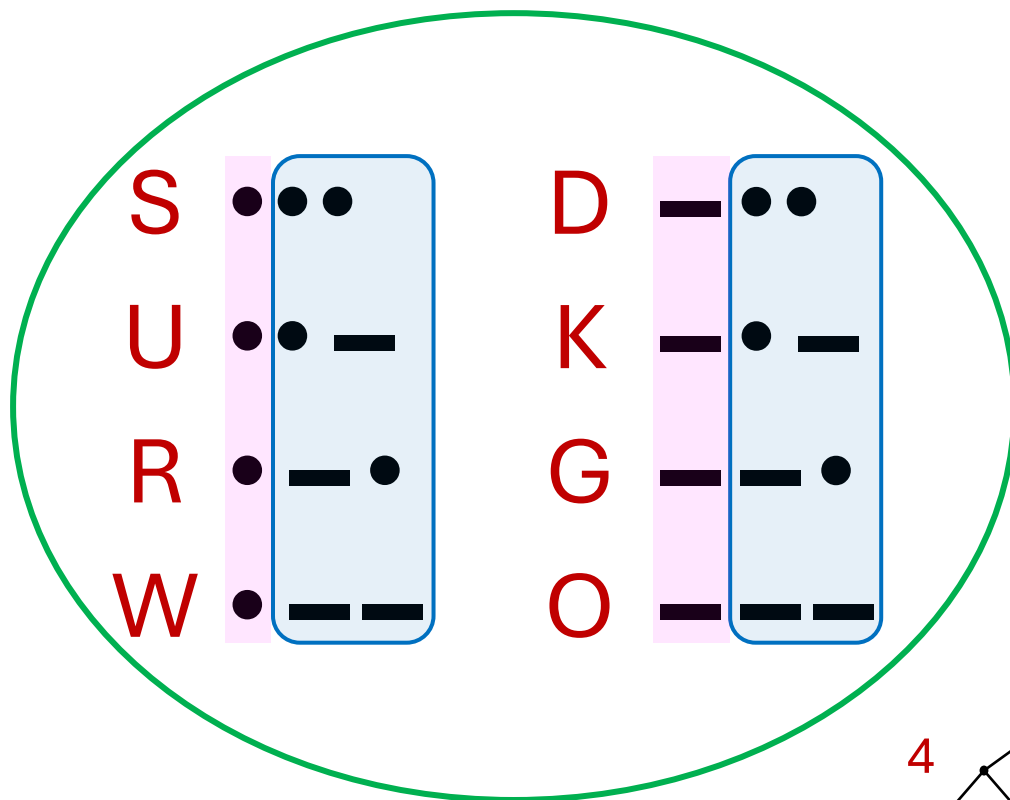
A • B •• C •••
D •••• E •••••
Z •••••
•••••
•••••
•••

I II III



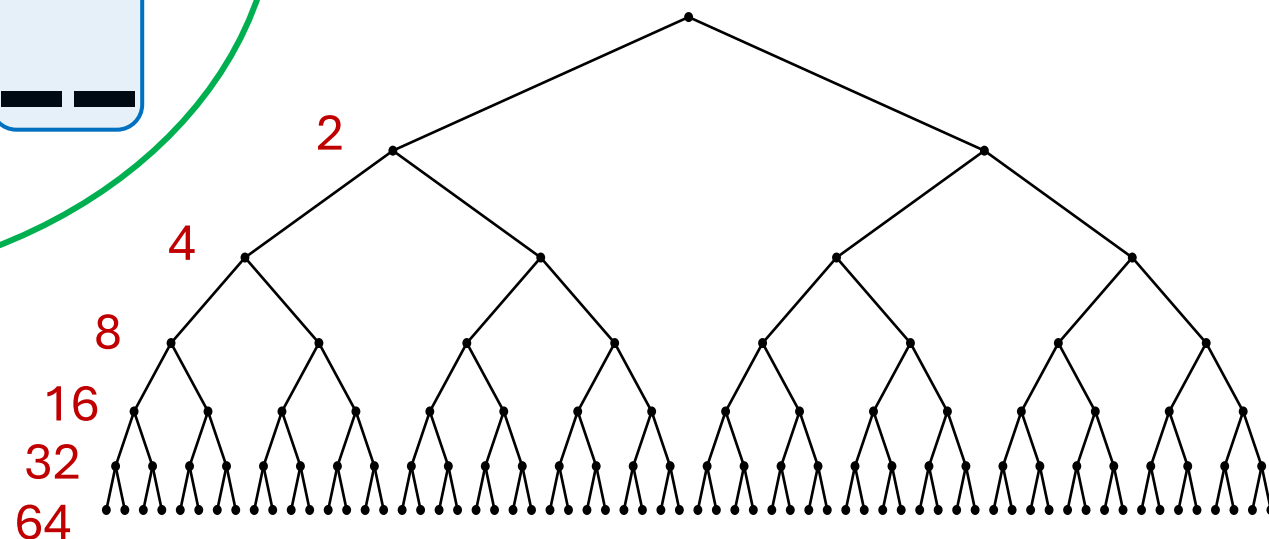


Morse code

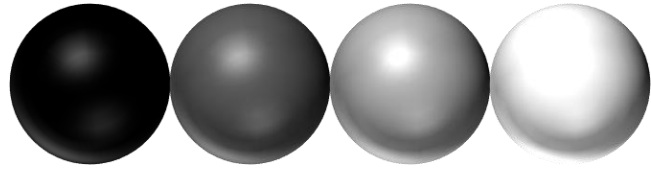


1 2
2 4
3 8
4 16

30
1,073,741,824

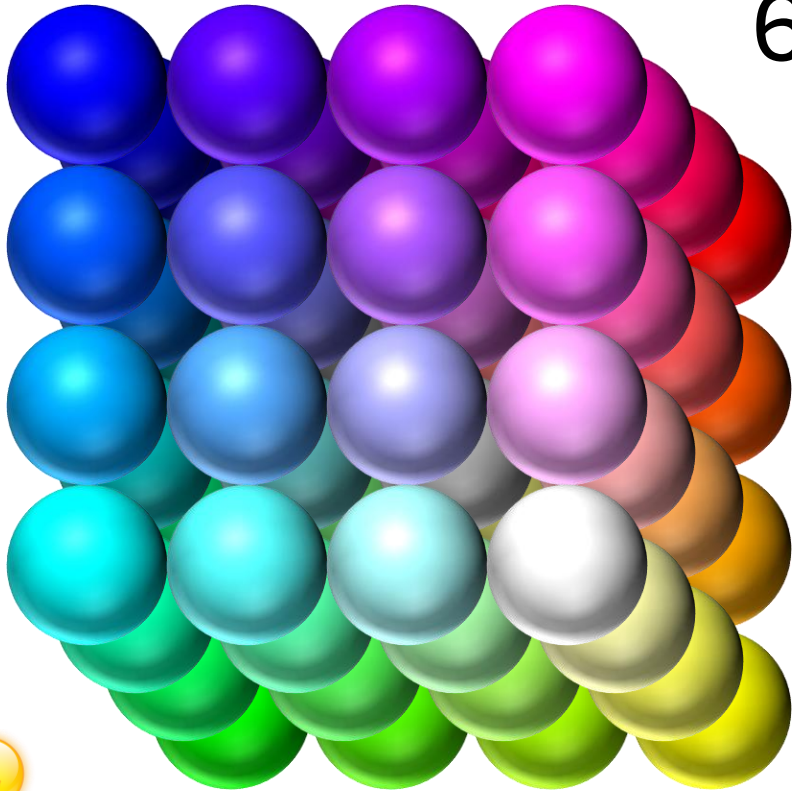


00 01 10 11



2 bits

6 bits



24 bits

?



16 million possible colours

森

0110 1000 1110 1110

β

0000 0011 1011 0010

ê

0000 0000 1110 1010

ℵ

0000 0101 1101 0000

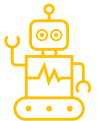
ふ

0011 0000 0111 0110

Unicode

16

65,536



Big numbers



intuition

Powers of 10

1,000,000

100,000

10,000

1,000

3rd power

10^3

$10 \times 10 \times 10$

100

2nd power

10^2

10

1st power

10^1

1



Powers of 10

1,000,000

100,000

10,000

1,000

100

10

1

3rd power

2nd power

1st power

10^3

10^2

10^1

1,000

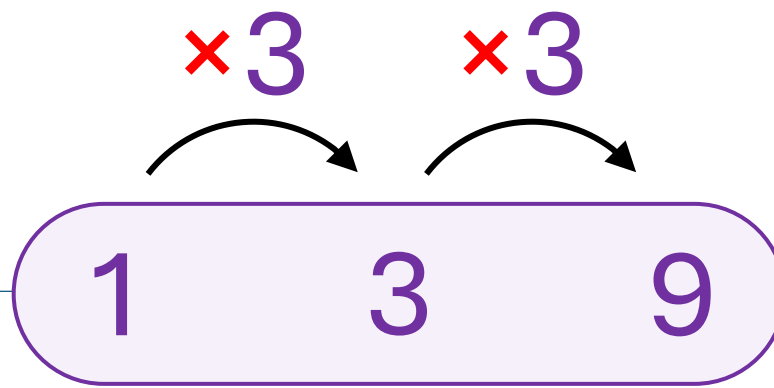
log

3



log scale

100,000



Richter scale
Decibels

10,000

exponent

exponential

log

1,000

3rd power

10³

1,000

3

100

2nd power

10²

10

1st power

1



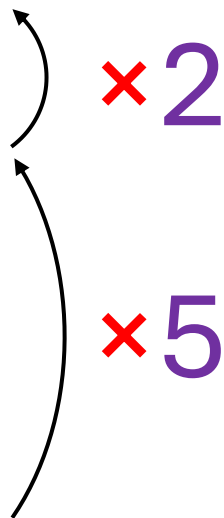
100

50

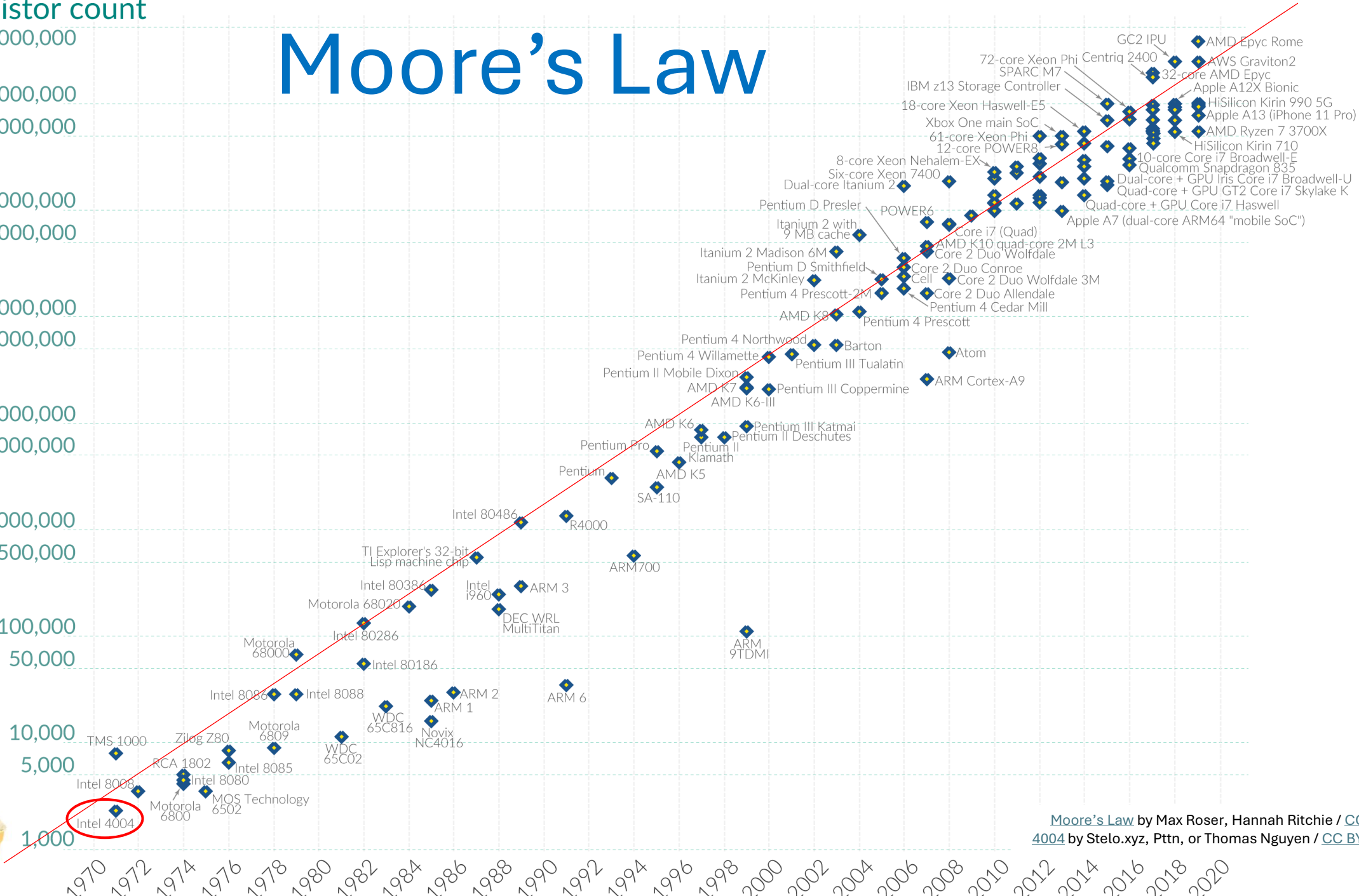
10

5

1

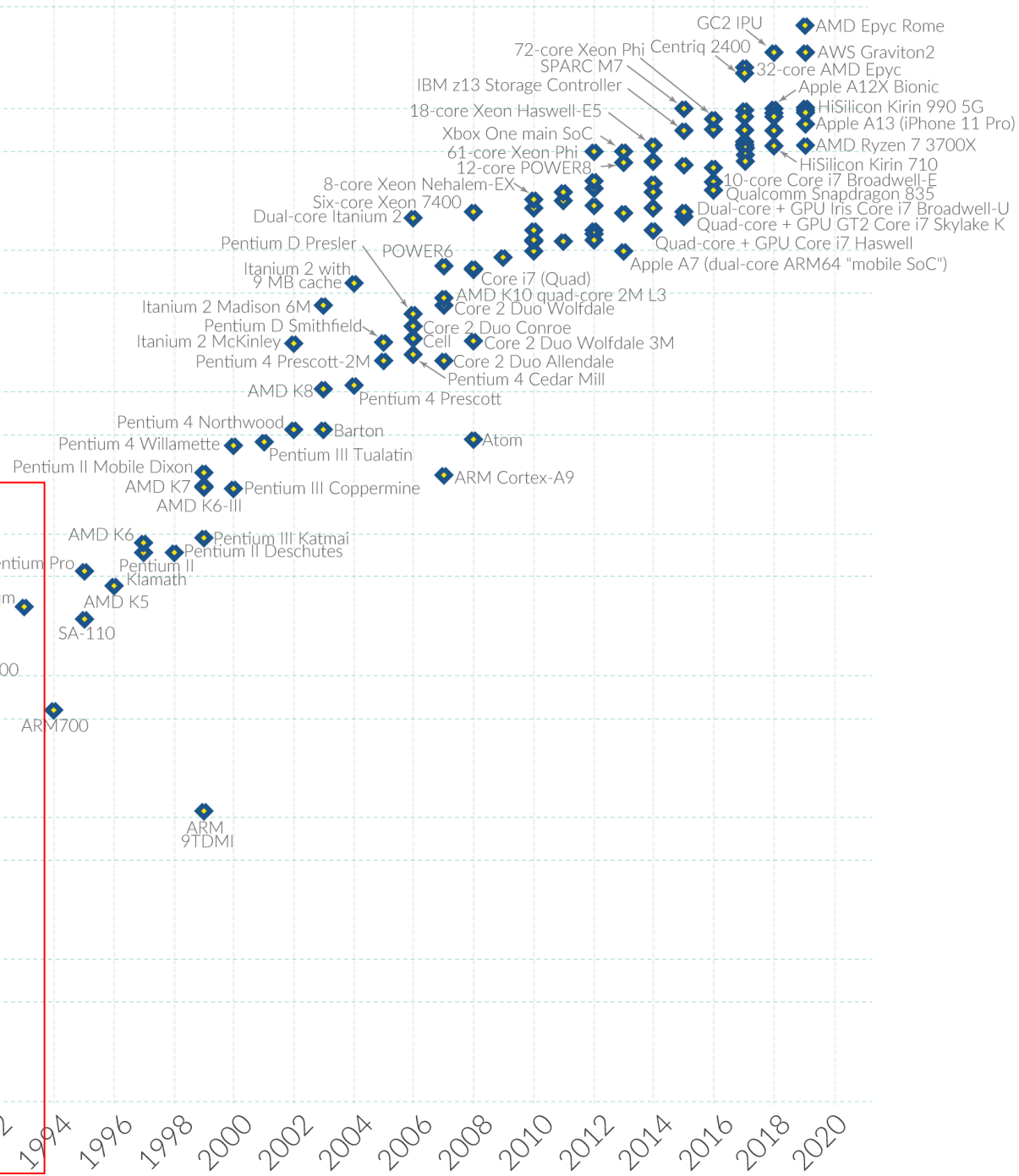


Transistor count



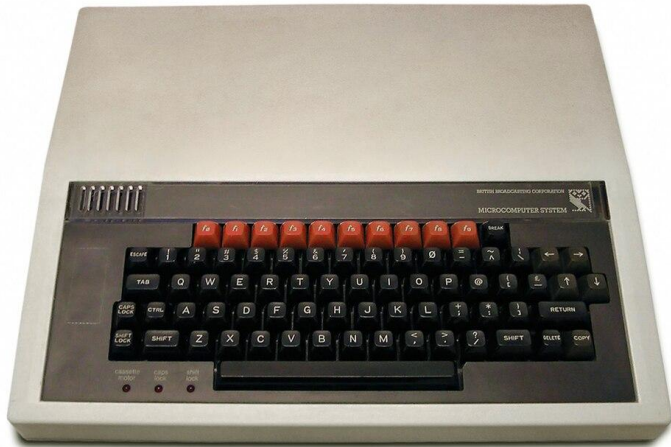
13

1,000



BBC Micro

1981



6502

Sinclair Spectrum

1982



Z80

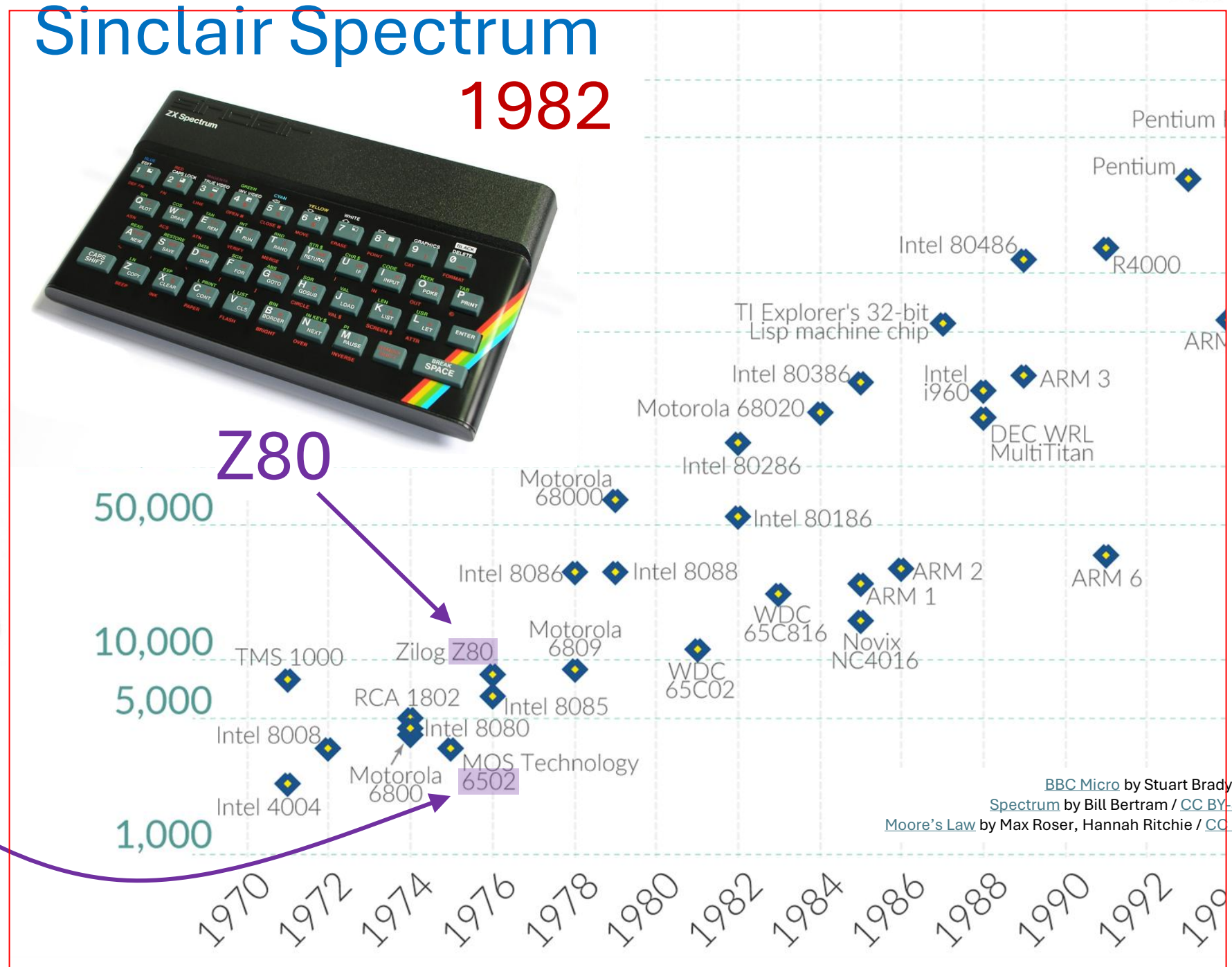
50,000

10,000

5,000

1,000

1970 1972 1974 1976 1978 1980 1982 1984 1986 1988 1990 1992 1994



BBC Micro by Stuart Brady / CC0
Spectrum by Bill Bertram / CC BY-SA 2.5
Moore's Law by Max Roser, Hannah Ritchie / CC BY 4.0



Transistor count

Moore's Law

50,000,000,000

10,000,000,000

5,000,000,000

1,000,000,000

500,000,000

100,000,000

50,000,000

10,000,000

5,000,000

1,000,000

500,000

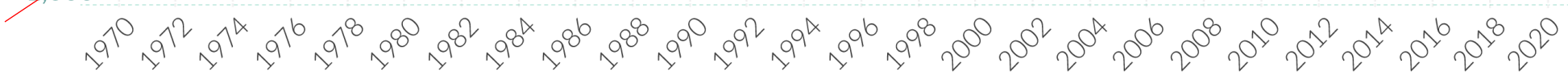
100,000

50,000

10,000

5,000

1,000

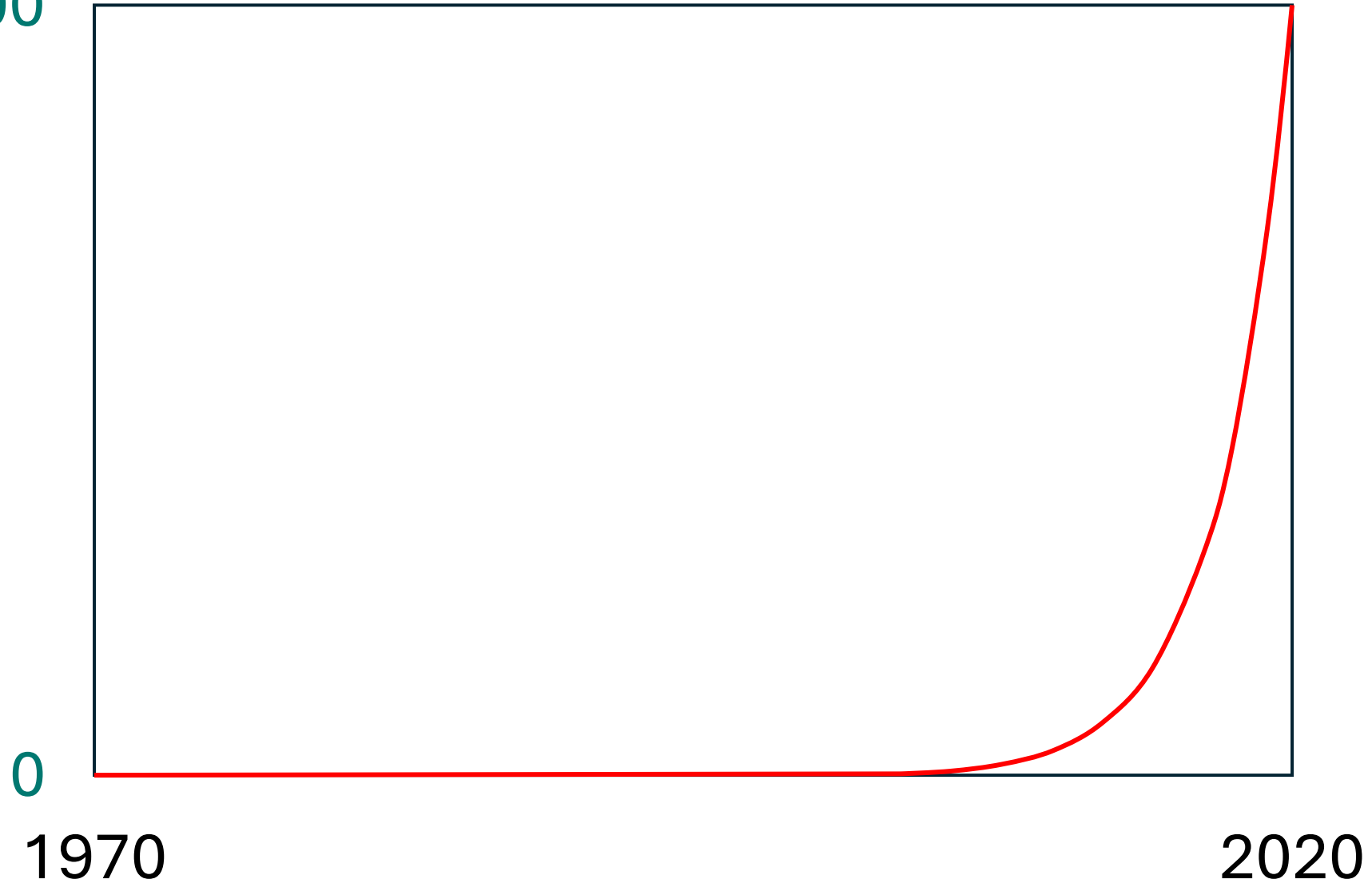


Our World
in Data

Moore's Law by Max Roser, Hannah Ritchie / CC BY 4.0

Transistor count

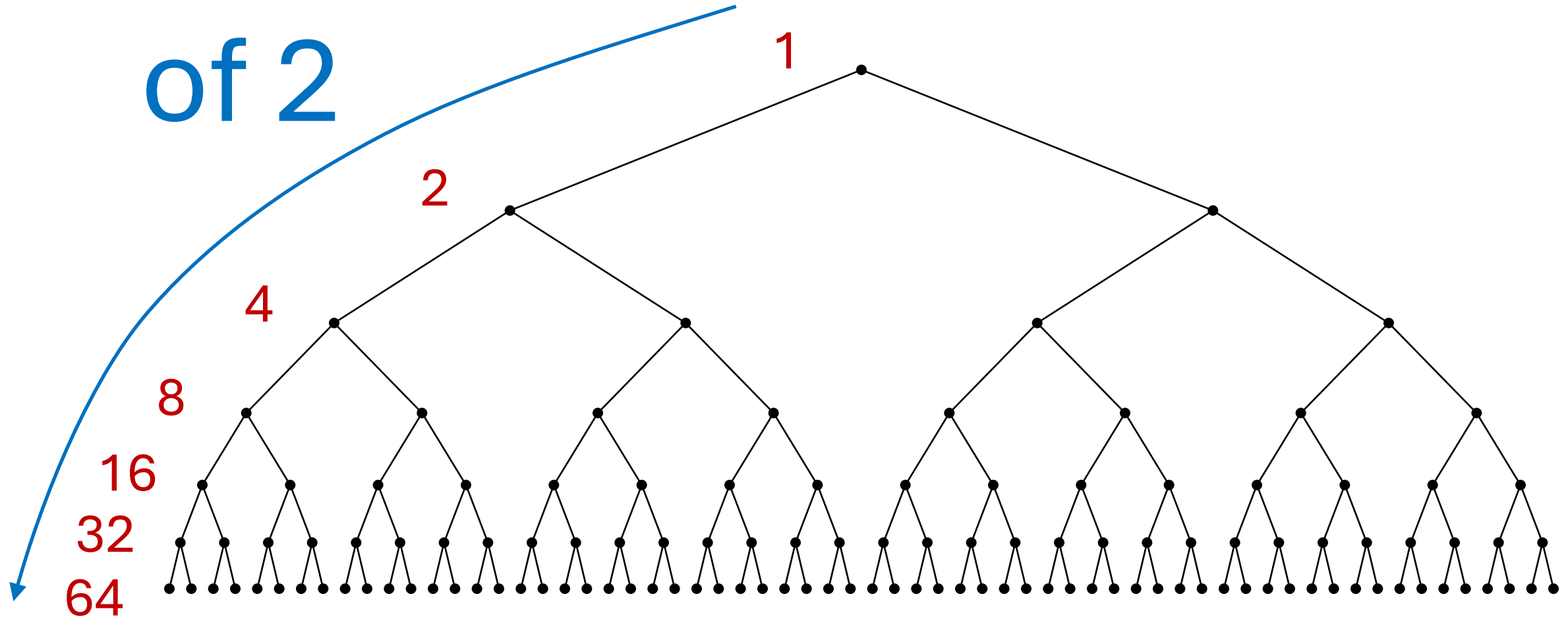
50,000,000,000

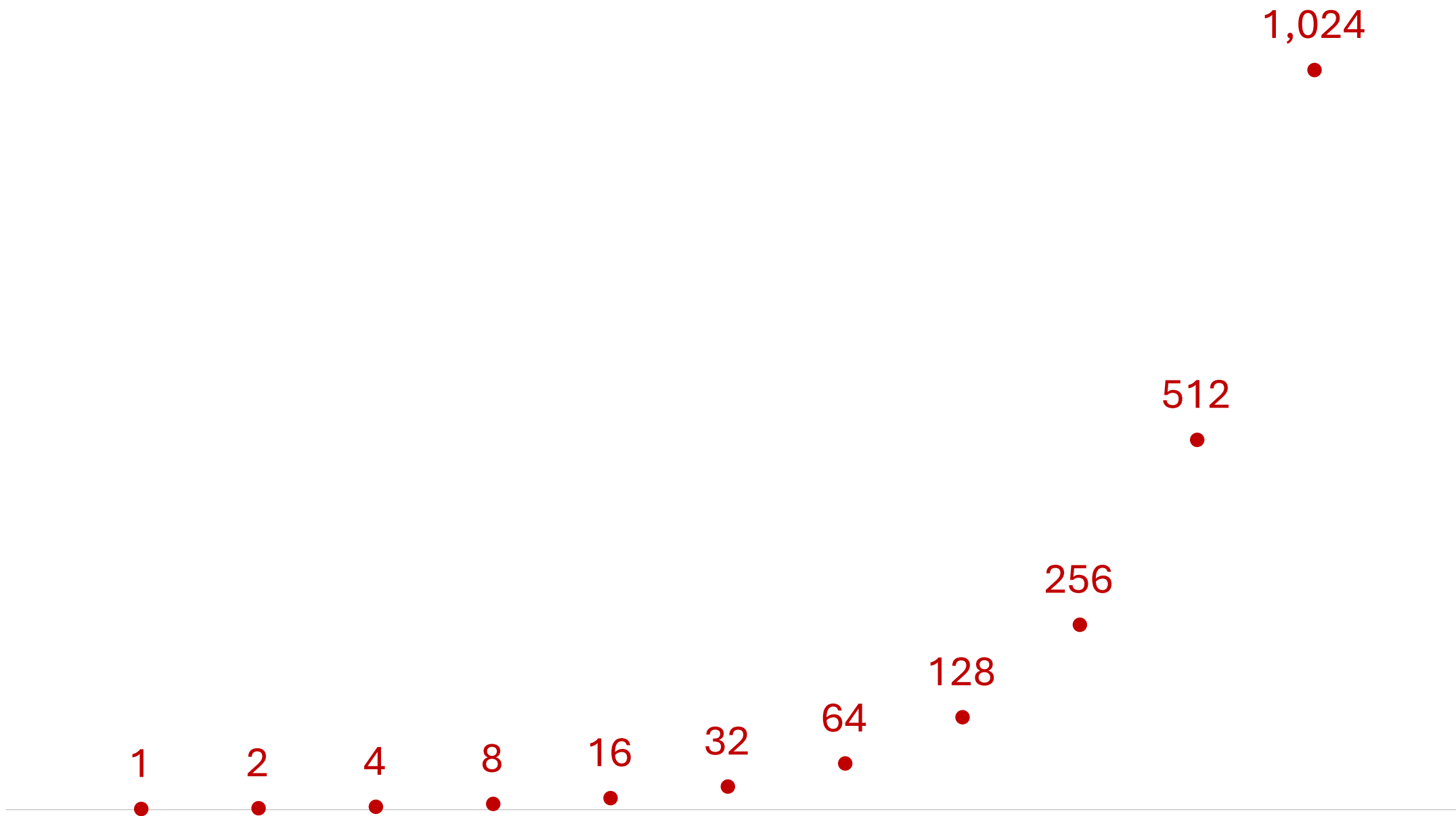


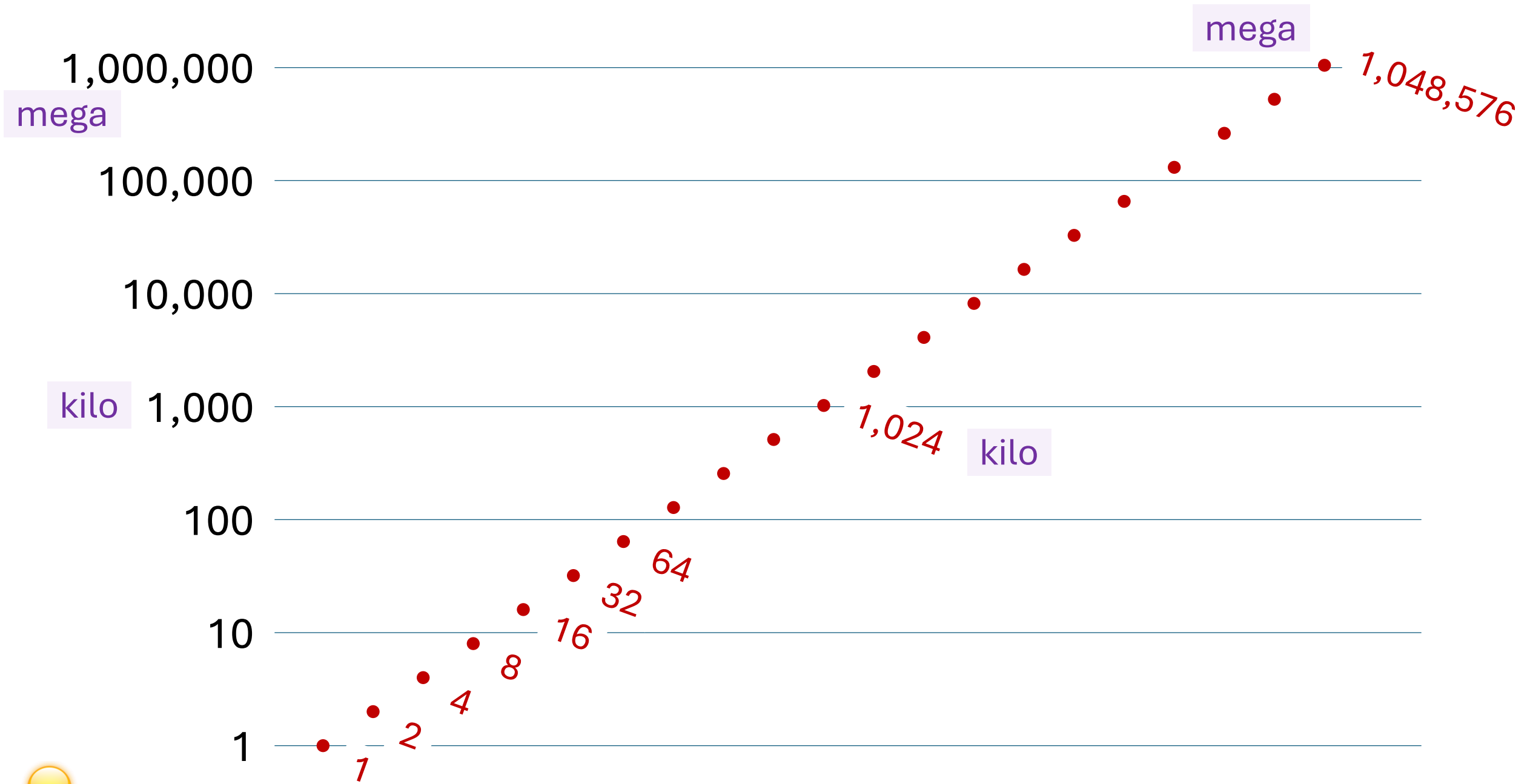
Moore's Law



powers of 2







This image shows a full page of blank, lined paper. It features approximately 20 horizontal blue lines spaced evenly across the page, typical of notebook paper. The lines are thin and light blue, set against a plain white background. There is no handwriting or other markings on the page.

quintillion

1,000,000,000,000,000,000

quadrillion

1,000,000,000,000,000

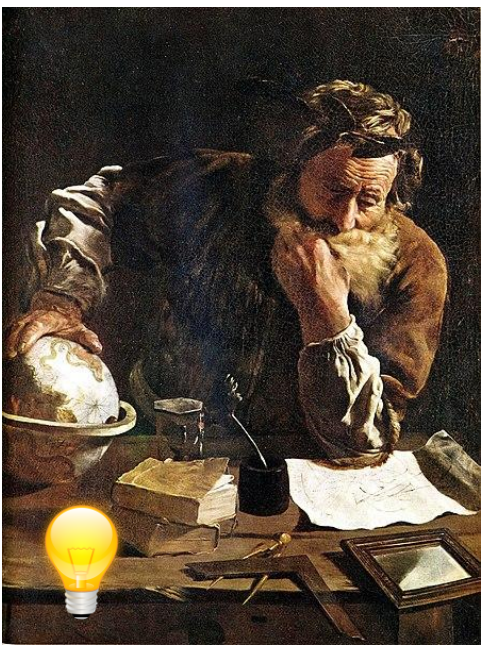
trillion

1,000,000,000,000

billion

1,000,000,000

Archimedes



— 10^{18}

— 10^{15} 10¹⁵

— 10^{12}

— 10^9

1,000,000

— 10^6 —————

1,000

— 10^3 — $10 \times 10 \times 10$ —————

1

— 10^0 —————

~~10 raised to the 15th power~~

10 to the 15

10 to the 15

floating-point

10^{18} operations per second (10^{18} flops)

+ - × ÷

2

3

3.65

36.5

22 MW

Human brain: 20 W



Cray-1

1975

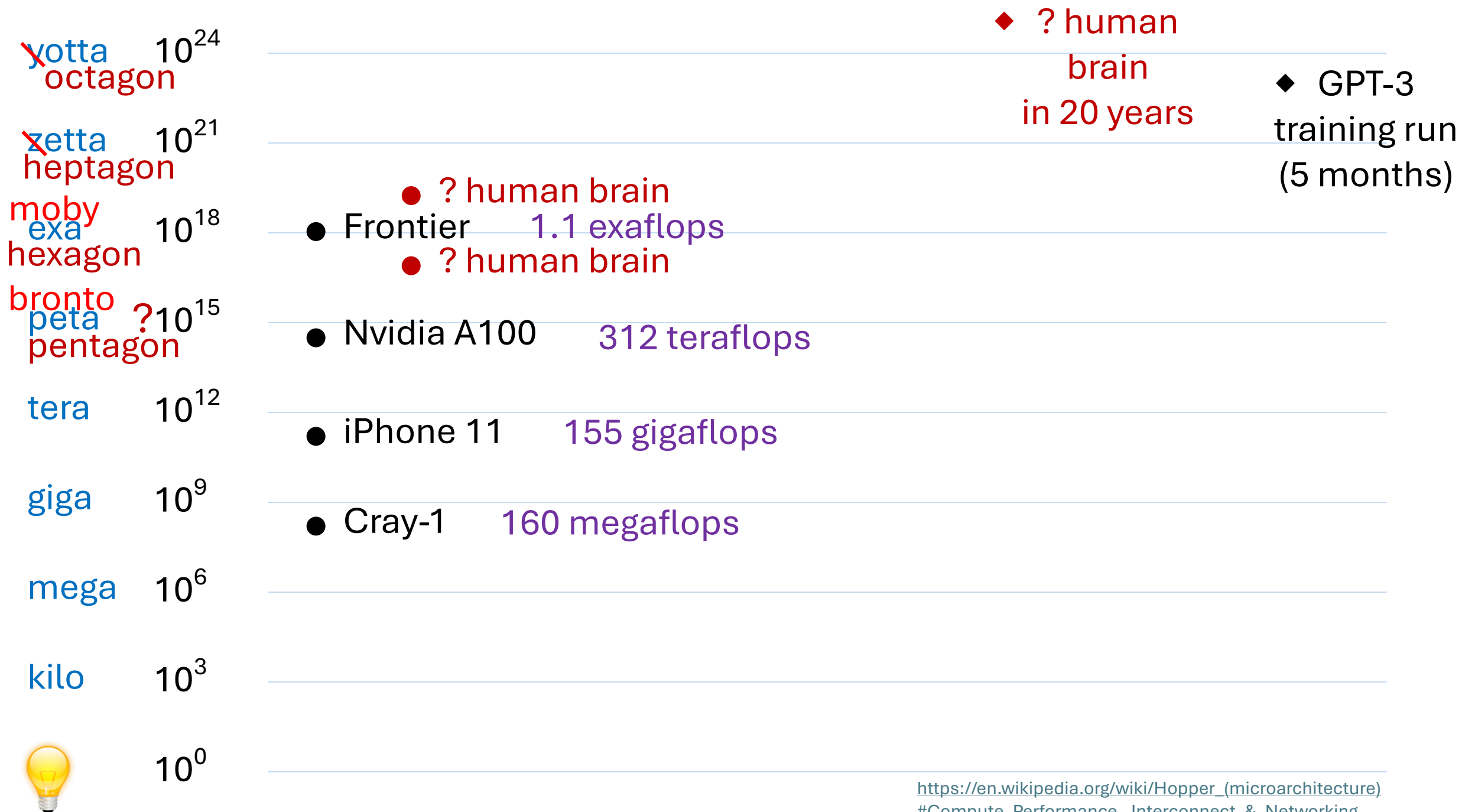
160 MFLOPS

6 *b*illion times
slower

115 kW

30 million times less
energy efficient





10^{160}

permutations of two packs of cards

10^{140}

10^{120}

the observable universe as a computer

10^{100}

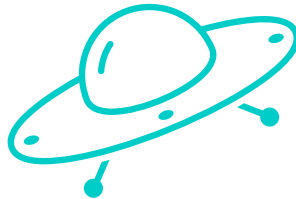
10^{80}

atoms in the observable universe

10^{60}

permutations of a pack of cards

10^{40}



10^{20}

stars in the observable universe

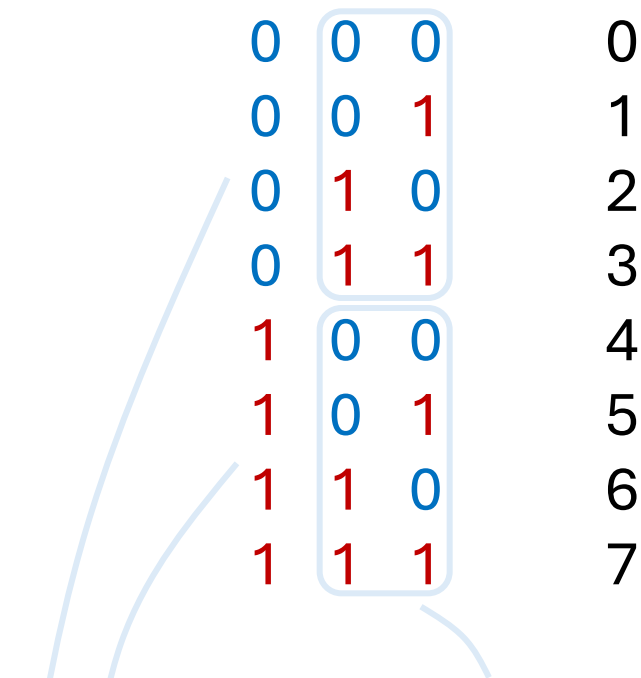
GPT-3 training run

all the computers for a year



1

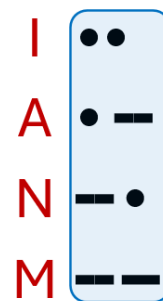
Binary numbers



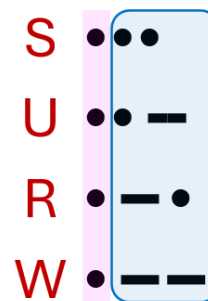
Binary



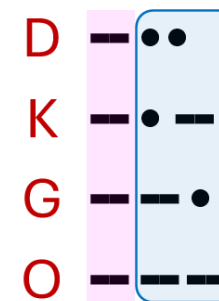
Leibniz



4



8



4	2	1	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Binary

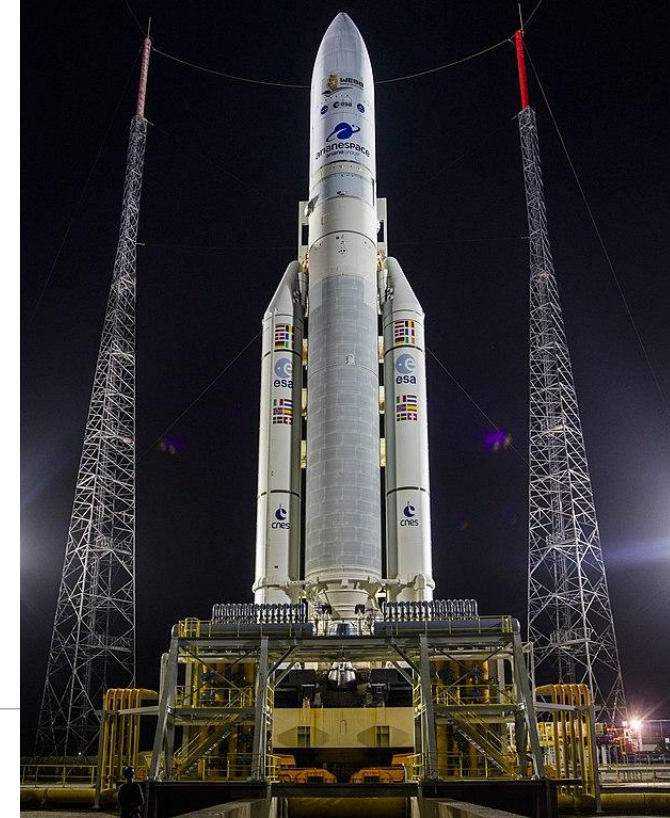
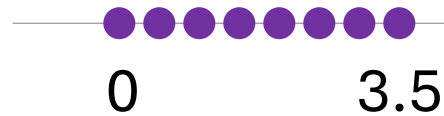
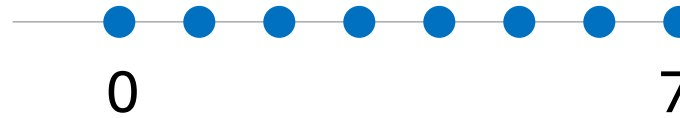


Leibniz



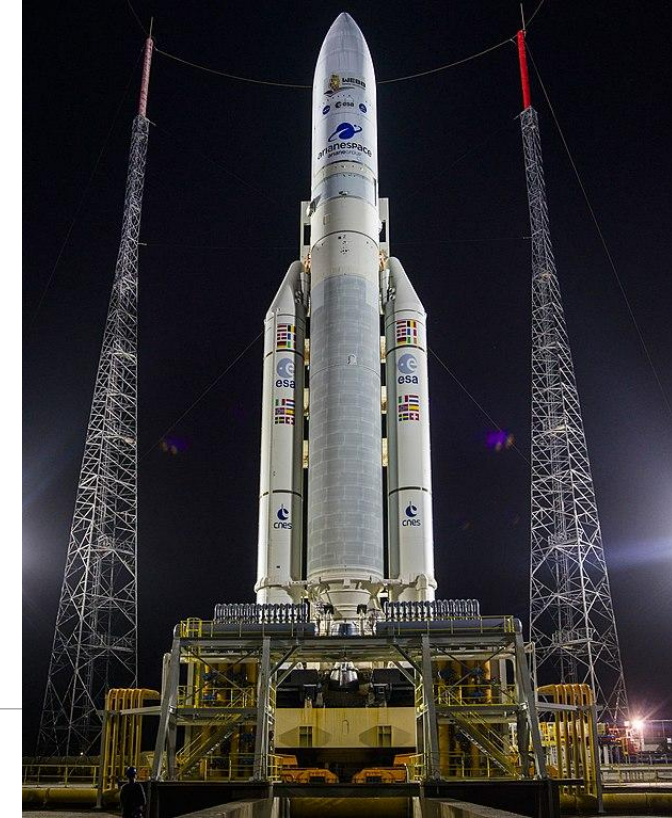
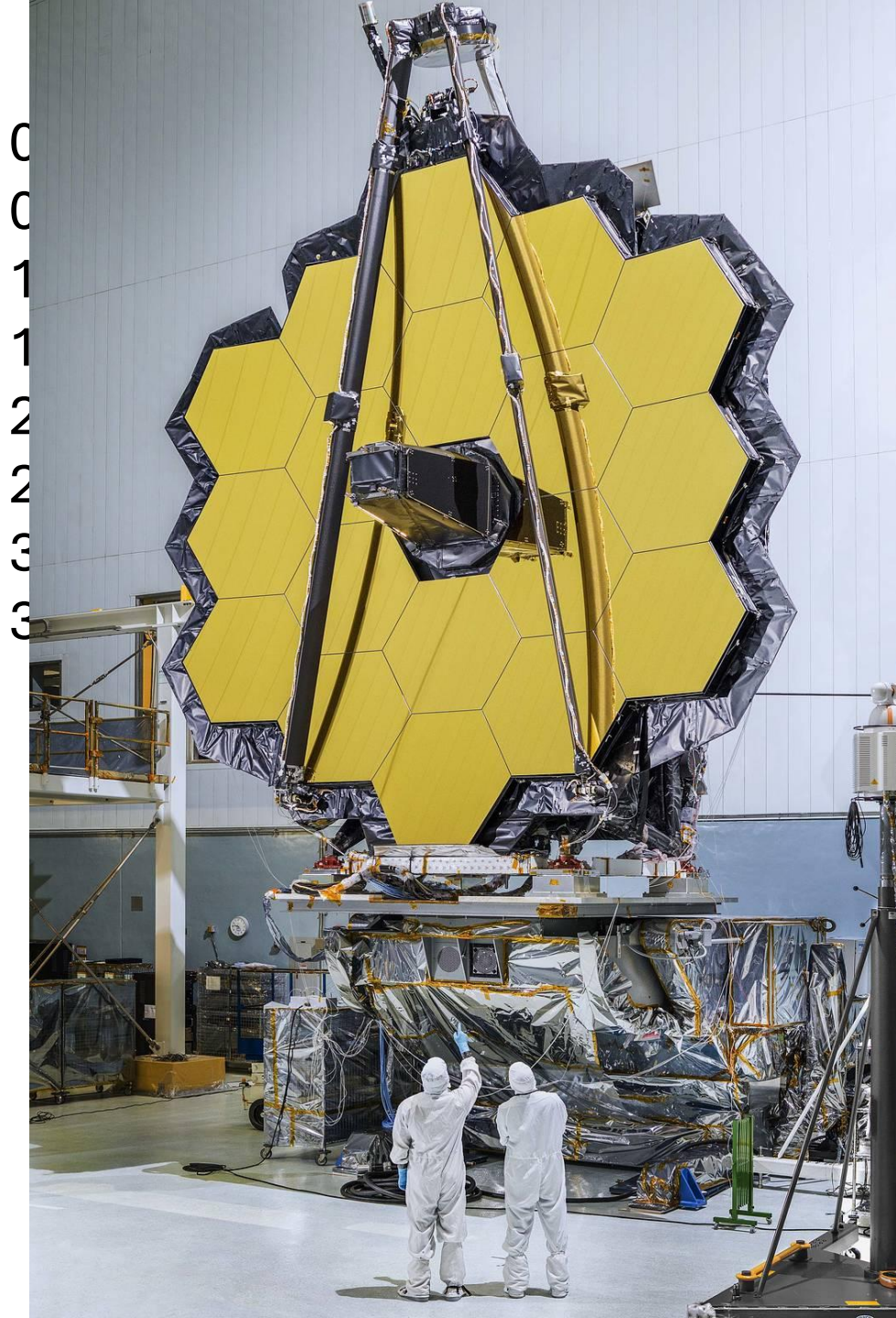
0	0	.	0	0
0	0	.	1	0.5
0	1	.	0	1
0	1	.	1	1.5
1	0	.	0	2
1	0	.	1	2.5
1	1	.	0	3
1	1	.	1	3.5

Fixed-point numbers



Ariane 5

0	0	.	0
0	0	.	1
0	1	.	0
0	1	.	1
1	0	.	0
1	0	.	1
1	1	.	0
1	1	.	1

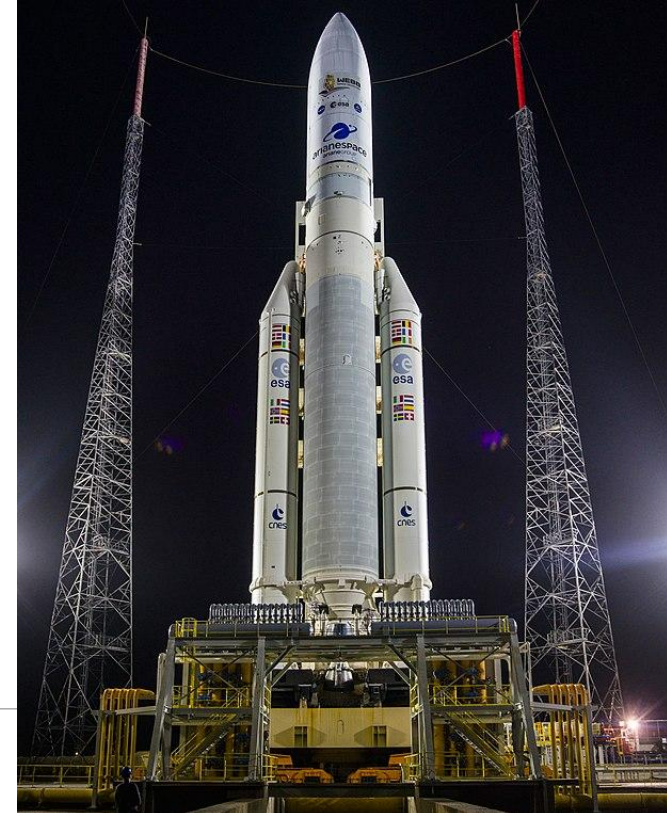
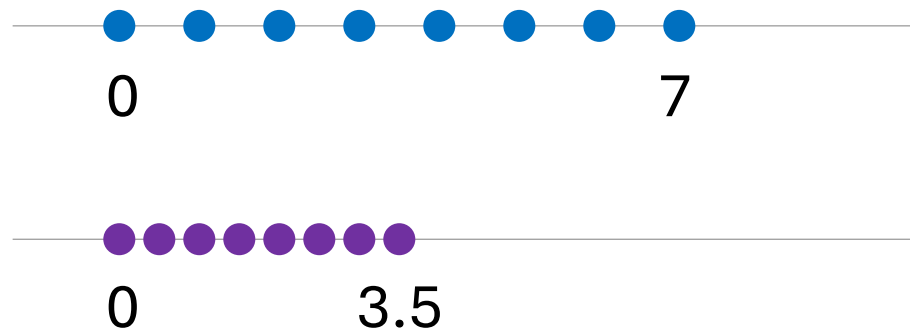


Ariane 5

0	0	.	0
0	0	.	1
0	1	.	0
0	1	.	1

0
0.5
1
1.5
2
2.5
3
3.5

Fixed-point numbers



Ariane 5

Negative numbers

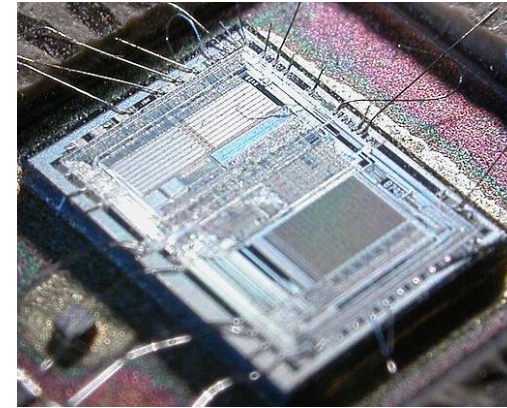
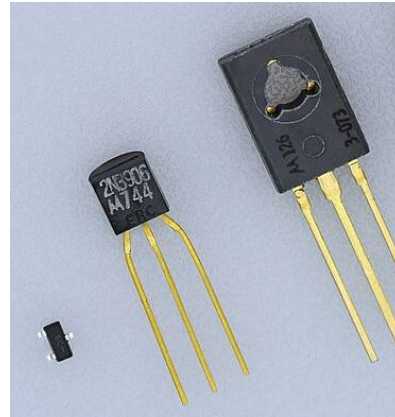
				Sign bit										
				↓										
0	0	0	0	+	{	0	0	0	0		0	0	0	0
0	0	1	1			0	0	1	1		0	0	1	1
0	1	0	2			0	1	0	2		0	1	0	2
0	1	1	3			0	1	1	3		0	1	1	3
1	0	0	4	-	{	1	0	0	-0		1	0	0	-4
1	0	1	5			1	0	1	-1		1	0	1	-3
1	1	0	6			1	1	0	-2		1	1	0	-2
1	1	1	7			1	1	1	-3		1	1	1	-1



Decimal



Binary

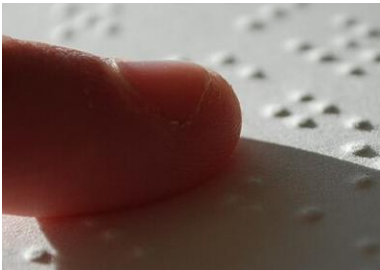
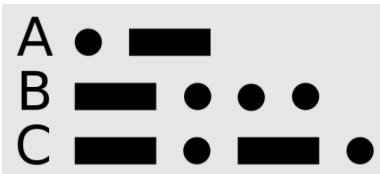


Digital

Transistor

Analog

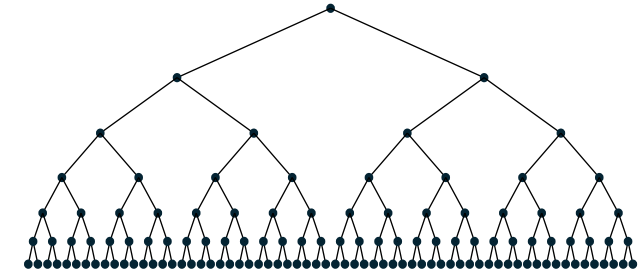
Binary



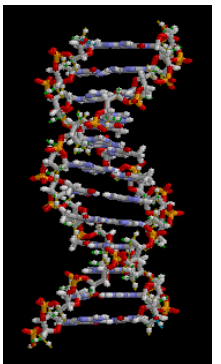
Ternary



Decimal



Digital things



[Money](#) by Avij

[DNA](#) by brian0918

[Dice](#) by Brett Jordan / [Unsplash](#)

[Cuneiform](#) by Unknown artist - Marie-Lan Nguyen

Binary

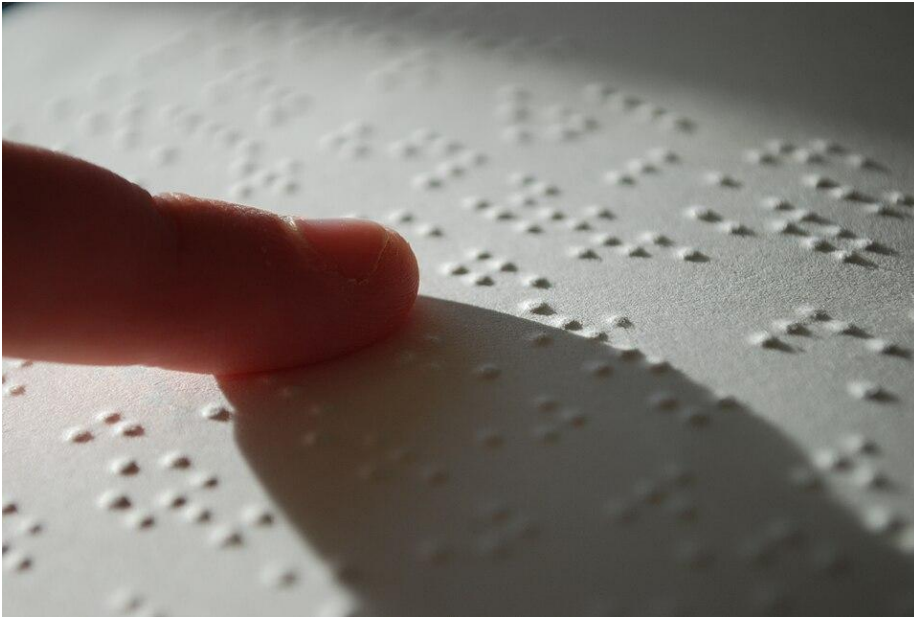
bit

on / off

0 / 1

yes / no

true / false



1824

A	●	■			
B	■	●	●	●	
C	■	●	■	●	
D	■	●	●		
E	●				

1837



logic

Logic

Transistor Two transistors
NOT gate

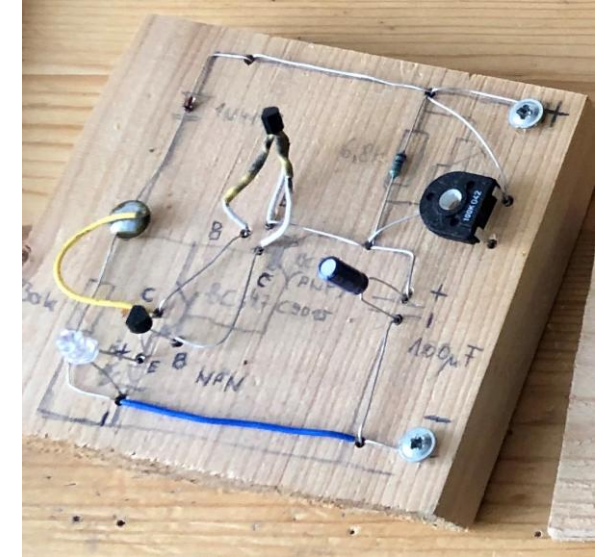
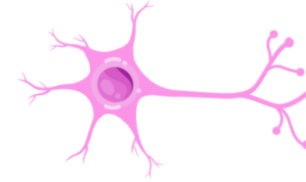
AND gate

AND gates

Making an AND gate

XOR gate OR gate

NAND gate



Arithmetic

Majority vote

Full 1-bit adder

3-bit adder

?

Round and round

Reset

Transistors

